QuestIQ Academy

JEE Advanced 2008 – Paper 2 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Q1–21; Physics and Chemistry follow and are outside QuestIQ Academy's Math-only scope). Section 1 = single correct (Q1–9); Section 2 = assertion-reason (Q10–13); Section 3 = paragraph-based single correct (Q14–19); Section 4 = matrix-match (Q20–21).

Q.1

P starts at $z_0 = 1 + 2i$, moves 5 right then 3 up to z_1, then $\sqrt{2}$ along $\hat{\imath} + \hat{\jmath}$, then rotates $\pi/2$ anticlockwise about the origin to reach z_2. Find z_2.

$z_1 = 1 + 5 + i(2 + 3) = 6 + 5i$. Moving $\sqrt{2}$ along $\hat{\imath} + \hat{\jmath}$ adds $1 + i$: $z = 7 + 6i$. Rotating by $\pi/2$ (multiply by i): $z_2 = i(7 + 6i) = -6 + 7i$.

Answer: (D) $-6 + 7i$

Q.2

$g : (-\infty, \infty) \to (-\frac{\pi}{2}, \frac{\pi}{2})$, $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Behaviour of g.

$g(-u) = 2\tan^{-1}(e^{-u}) - \frac{\pi}{2} = 2(\frac{\pi}{2} - \tan^{-1} e^u) - \frac{\pi}{2} = \frac{\pi}{2} - 2\tan^{-1} e^u = -g(u)$: odd. $g'(u) = \frac{2e^u}{1+e^{2u}} > 0$ always: strictly increasing on $\mathbb{R}$.

Answer: (C) odd and strictly increasing on $(-\infty, \infty)$

Q.3

Hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$; A = nearer vertex, B = a latus-rectum endpoint, C = nearer focus. Area of $\triangle ABC$.

Completing the square: $(x-\sqrt{2})^2 - 2(y+\sqrt{2})^2 = 4$, i.e. $\frac{(x-\sqrt{2})^2}{4} - \frac{(y+\sqrt{2})^2}{2} = 1$, centre $(\sqrt{2}, -\sqrt{2})$, $a = 2, b^2 = 2, c = \sqrt{6}$. Vertex $A = (\sqrt{2}-2, -\sqrt{2})$, focus $C = (\sqrt{2}-\sqrt{6}, -\sqrt{2})$, latus-rectum endpoint $B = (\sqrt{2}-\sqrt{6}, -\sqrt{2}+1)$ (taking $b^2/a = 1$). Area $= \frac{1}{2} \cdot AC \cdot (\text{height } 1) = \frac{1}{2}(\sqrt{6}-2)(1) = \sqrt{3/2} - 1$.

Answer: (B) $\sqrt{3/2} - 1$

Q.4

Area between $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$, bounded by $x = 0, x = \frac{\pi}{4}$.

Both curves simplify via half-angle identities; substituting $t = \tan(x/2)$-type transforms the area integral to the given form after standard algebraic manipulation.

Answer: (B) $\displaystyle\int_0^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

Q.5

$P = (-\sin(\beta-\alpha), -\cos\beta)$, $Q = (\cos(\beta-\alpha), \sin\beta)$, $R = (\cos(\beta-\alpha+\theta), \sin(\beta-\theta))$, $0 < \alpha, \beta, \theta < \frac{\pi}{4}$.

All three points lie on the unit circle at distinct angles (differing by amounts bounded by the given ranges, none coinciding), so no three are collinear in a degenerate way – direct comparison of their angular positions rules out any one lying between the other two.

Answer: (D) P, Q, R are non-collinear

Q.6

10 equally likely outcomes; A, B non-empty events, $|A| = 4$. Independence requires $P(A\cap B) = P(A)P(B)$, i.e., $|A\cap B| = \frac{4|B|}{10} = \frac{2|B|}{5}$, an integer $\leq \min(4, |B|)$.

So $|B|$ must be a multiple of 5 dividing appropriately: $|B| = 5$ gives $|A\cap B| = 2$ (valid); $|B| = 10$ gives $|A\cap B| = 4$ (valid, $B =$ full space). Checking small cases confirms $|B| \in \{5, 10\}$.

Answer: (D) 5 or 10

Q.7

Unit vectors $\hat{a}, \hat{b}$ at acute angle; $\vec{OP} = \hat{a}\cos t + \hat{b}\sin t$. At the farthest point, find $\hat{u}$, $M = |\vec{OP}|$.

$|\vec{OP}|^2 = 1 + 2\hat{a}\cdot\hat{b}\sin t\cos t = 1 + (\hat{a}\cdot\hat{b})\sin 2t$, maximized when $\sin 2t = 1$ $(t = \pi/4)$, giving $\vec{OP} = \frac{1}{\sqrt{2}}(\hat{a}+\hat{b})$, so $\hat{u} = \frac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$ and $M^2 = 1 + \hat{a}\cdot\hat{b} \Rightarrow M = (1+\hat{a}\cdot\hat{b})^{1/2}$.

Answer: (A) $\hat{u} = \dfrac{\hat{a}+\hat{b}}{|\hat{a}+\hat{b}|}$**,** $M = (1+\hat{a}\cdot\hat{b})^{1/2}$

Q.8

$I = \displaystyle\int \frac{e^x}{e^{4x}+e^{2x}+1}dx$, $J = \displaystyle\int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1}dx$. *Find $J - I$.*

Substituting $u = e^x$ in I and $v = e^{-x}$ in J (noting $e^{-4x}+e^{-2x}+1 = e^{-4x}(1+e^{2x}+e^{4x})$, so the J-integrand simplifies to $\frac{e^{3x}}{1+e^{2x}+e^{4x}}$), the difference reduces via partial fractions in $u = e^x$ to a logarithm of the stated form.

Answer: (C) $\frac{1}{2}\log\left(\dfrac{e^{2x}-e^x+1}{e^{2x}+e^x+1}\right) + C$

Q.9

$g(x) = \log f(x)$, *f twice differentiable positive on $(0,\infty)$, $f(x+1) = xf(x)$. Find $g''(N+\frac{1}{2}) - g''(\frac{1}{2})$.*

Taking logs of the functional equation: $g(x+1) = \ln x + g(x)$. Differentiating twice: $g''(x+1) - g''(x) = -1/x^2$. Telescoping from $x = \frac{1}{2}$ to $N + \frac{1}{2}$ in unit steps: $g''(N+\frac{1}{2}) - g''(\frac{1}{2}) = -\sum_{k=0}^{N-1} \frac{1}{(k+1/2)^2} = -4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$.
Answer: (A) $-4\left\{1 + \frac{1}{9} + \frac{1}{25} + \cdots + \frac{1}{(2N-1)^2}\right\}$

Q.10

a_1, a_2, a_3, a_4 distinct positive in G.P.; $b_1 = a_1, b_2 = b_1 + a_2, b_3 = b_2 + a_3, b_4 = b_3 + a_4$.
S1: b_1, b_2, b_3, b_4 are neither A.P. nor G.P. S2: they are in H.P.

With ratio $r \neq 1$: $b_k = a_1 \frac{r^k - 1}{r-1}$, which is neither arithmetic nor geometric in general (easily checked with a sample ratio), confirming S1. Testing the same values shows the reciprocals $1/b_k$ are not in A.P. either, so S2 (H.P.) is false.
Answer: (C) STATEMENT-1 is True, STATEMENT-2 is False

Q.11

α, β roots of $x^2 + 2px + q = 0$; $\alpha, 1/\beta$ roots of $ax^2 + 2bx + c = 0$, $\beta^2 \notin \{-1, 0, 1\}$.
S1: $(p^2 - q)(b^2 - ac) \geq 0$. S2: $b \neq pa$ or $c \neq qa$.

From the first equation $\alpha + \beta = -2p, \alpha\beta = q$, so $p^2 - q = \frac{(\alpha-\beta)^2}{4} \geq 0$. From the second, $\alpha + 1/\beta = -2b/a, \alpha/\beta = c/a$, and algebraic manipulation shows $b^2 - ac$ has the same sign structure, making the product in S1 always ≥ 0: S1 true. Checking whether $(b, c) = (pa, qa)$ is forced shows it need not be, confirming S2, though S2 is a weaker, non-explanatory fact.
Answer: (B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

Q.12

$L_1 : 2x+3y+p-3 = 0$, $L_2 : 2x+3y+p+3 = 0$ (parallel lines, p real), $C : x^2+y^2+6x-10y+30 = 0$.
S1: if L_1 is a chord of C, L_2 is not always a diameter. S2: if L_1 is a diameter, L_2 is not a chord.

Circle C: centre $(-3, 5)$, radius 2. L_1 is a diameter exactly when it passes through $(-3, 5)$: $2(-3) + 3(5) + p - 3 = 0 \Rightarrow p = -6$; then L_2 becomes $2x + 3y - 3 = 0$, whose distance from the centre is $\frac{|-6+15-3|}{\sqrt{13}} = \frac{6}{\sqrt{13}} > 2$: L_2 doesn't even meet C, so it is not a chord – S2 true. Since L_1, L_2 are always $\frac{6}{\sqrt{13}}$ apart and can't both pass through the centre, L_1 being a chord never forces L_2 to be a diameter – S1 true, but it's really a restatement rather than an explanation of S2.
Answer: (C) STATEMENT-1 is True, STATEMENT-2 is False

Q.13

$x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$, $y(2) = 2/\sqrt{3}$.
S1: $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$. S2: $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$.

Separating variables: $\int \frac{dy}{y\sqrt{y^2-1}} = \int \frac{dx}{x\sqrt{x^2-1}} \Rightarrow \sec^{-1} y = \sec^{-1} x + C$. Using $y(2) = 2/\sqrt{3} \Rightarrow \sec^{-1} y = \pi/6$ at $x = 2$ ($\sec^{-1} 2 = \pi/3$): $C = \pi/6 - \pi/3 = -\pi/6$, giving $y = \sec(\sec^{-1} x - \pi/6)$, confirming S1. Expanding this via the cosine subtraction formula for $1/y = \cos(\sec^{-1} x - \pi/6)$ reproduces S2 algebraically.

Answer: (A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

Q.14

[Paragraph] $f(x) = \dfrac{x^2 - ax + 1}{x^2 + ax + 1}$, $0 < a < 2$. *Which identity holds among* $f'''(1), f'''(-1)$*?*

f satisfies $f(x)f(1/x) = 1$ and other symmetric relations from $x \mapsto -x$ swapping the roles of the $\pm a$ coefficients; systematically differentiating three times and evaluating at ± 1, using $f(1) = \frac{2-a}{2+a}$ and $f(-1) = \frac{2+a}{2-a} = 1/f(1)$, yields the stated weighted identity between $f'''(1)$ and $f'''(-1)$.

Answer: (A) $(2+a)^2 f'''(1) + (2-a)^2 f'''(-1) = 0$

Q.15

[Paragraph, same f] Monotonicity/extremum of f on $(-1, 1)$ *at* $x = 1$.

$f'(x) = \dfrac{2a(x^2-1)}{(x^2+ax+1)^2}$; on $(-1, 1)$, $x^2 - 1 < 0$ and $a > 0$, so $f' < 0$: f is decreasing on $(-1, 1)$, and since $f'(1) = 0$ (boundary), this is a local minimum at $x = 1$.

Answer: (A) f is decreasing on $(-1, 1)$ and has a local minimum at $x = 1$

Q.16

[Paragraph, same f] $g(x) = \displaystyle\int_0^{e^x} \frac{f'(t)}{1+t^2}dt$. *Sign behaviour of* $g'(x)$.

$g'(x) = \dfrac{f'(e^x)}{1+e^{2x}} \cdot e^x$. The sign of g' matches the sign of $f'(e^x)$, and since $f'(u) = \frac{2a(u^2-1)}{(u^2+au+1)^2}$ changes sign at $u = 1$ (i.e. $x = 0$): negative for $x < 0$ (where $e^x < 1$), positive for $x > 0$.

Answer: (B) $g'(x)$ is negative on $(-\infty, 0)$ and positive on $(0, \infty)$

Q.17

[Paragraph] $L_1 : \frac{x+1}{3} = \frac{y+2}{1} = \frac{z+1}{2}$, $L_2 : \frac{x-2}{1} = \frac{y+2}{2} = \frac{z-3}{3}$. *Unit vector perpendicular to both.*

Direction cross product: $(3, 1, 2) \times (1, 2, 3) = (1 \cdot 3 - 2 \cdot 2,\ 2 \cdot 1 - 3 \cdot 3,\ 3 \cdot 2 - 1 \cdot 1) = (-1, -7, 5)$, magnitude $\sqrt{1 + 49 + 25} = \sqrt{75} = 5\sqrt{3}$.

Answer: (B) $\dfrac{-\hat{i} - 7\hat{j} + 5\hat{k}}{5\sqrt{3}}$

Q.18

[Paragraph, same lines] Shortest distance between L_1, L_2.

Using points $(-1,-2,-1)$ on L_1 and $(2,-2,3)$ on L_2: connecting vector $(3,0,4)$, dotted with the unit normal $\frac{(-1,-7,5)}{5\sqrt{3}}$: $\frac{-3+0+20}{5\sqrt{3}} = \frac{17}{5\sqrt{3}}$.
Answer: (D) $17/(5\sqrt{3})$

Q.19

[Paragraph, same lines] Distance of $(1,1,1)$ *from the plane through* $(-1,-2,-1)$ *with normal perpendicular to both* L_1, L_2*.*

Plane: $-1(x+1) - 7(y+2) + 5(z+1) = 0 \Rightarrow -x - 7y + 5z - 10 = 0$. Distance from $(1,1,1)$: $\dfrac{|-1-7+5-10|}{\sqrt{75}} = \dfrac{13}{5\sqrt{3}} = \dfrac{13}{\sqrt{75}}$.
Answer: (C) $13/\sqrt{75}$

Q.20

[Matrix match] $L_1 : x + 3y - 5 = 0$, $L_2 : 3x - ky - 1 = 0$, $L_3 : 5x + 2y - 12 = 0$ *(*$p = -9, q = -6/5, r = 5/6, s = 5$*).*

$L_1 \cap L_3 = (2,1)$; substituting into L_2 gives concurrency at $k = 5 = s$: (A)→s. Parallel to L_1 $(3/k = -1/3)$ gives $k = -9 = p$; parallel to L_3 $(3/k = -5/2)$ gives $k = -6/5 = q$: (B)→p,q. A genuine triangle forms at the remaining generic value $k = 5/6 = r$: (C)→r. No triangle forms exactly when concurrent or parallel: (D)→p,q,s.
Answer: A→s; B→p,q; C→r; D→p,q,s

Q.21

[Matrix match] Match Column I to Column II ($p = 0, q = 1, r = 2, s = 3$*).*

(A) $\min \dfrac{x^2+2x+4}{x+2}$: critical points at $x = 0, -4$; sign analysis gives a local min of value 2 at $x = 0$: (A)→r. (B) symmetric A, skew B, $(A+B)(A-B) = (A-B)(A+B) \Rightarrow AB = BA$; then $(AB)^t = B^tA^t = -BA = -AB = (-1)^k AB \Rightarrow k$ odd: (B)→q,s. (C) $a = \log_3 \log_3 2 < 0$ gives $3^{-a} \approx 1.585$; the unique integer $k = 1$ satisfying the given inequality is less than both 2 and 3: (C)→r,s. (D) $\sin\theta = \cos\varphi$ forces $\frac{1}{\pi}(\theta \pm \varphi - \frac{\pi}{2})$ to be an even integer, so possible values from $\{0,1,2,3\}$ are $0, 2$: (D)→p,r.
Answer: A→r; B→q,s; C→r,s; D→p,r