QuestIQ Academy

JEE Advanced 2011 – Paper 1 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = single correct; Section 2 = one or more correct; Section 3 = paragraph-based single correct.

Q.1

$(2x)^{\ln 2} = (3y)^{\ln 3}$, $3^{\ln x} = 2^{\ln y}$. *Find* x_0.

Taking logs of both equations and eliminating $\ln y$ using $\ln y = \frac{\ln 3}{\ln 2} \ln x$ leads, after simplification, to $-\ln 2 = \ln x$, so $x = 1/2$.

Answer: (C) $1/2$

Q.2

$$\int_{\sqrt{\ln 2}}^{\sqrt{\ln 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ln 6 - x^2)} dx.$$

Since $(\sqrt{\ln 2})^2 + (\sqrt{\ln 3})^2 = \ln 6$, the substitution $t^2 = \ln 6 - x^2$ swaps the limits and shows the integral I equals the "companion" integral I' with numerator $\sin(\ln 6 - x^2)$; since $I + I' = \int x\,dx = \frac{1}{2}(\ln 3 - \ln 2)$ and $I = I'$, we get $I = \frac{1}{4} \ln \frac{3}{2}$.

Answer: (A) $\frac{1}{4} \ln \frac{3}{2}$

Q.3

$\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. *Vector* $\vec{v}$ *in the plane of* $\vec{a}, \vec{b}$ *with projection* $1/\sqrt{3}$ *on* $\vec{c}$.

Writing $\vec{v} = \alpha\vec{a} + \beta\vec{b} = (\alpha + \beta, \alpha - \beta, \alpha + \beta)$, the projection condition $\vec{v} \cdot \vec{c}/|\vec{c}| = 1/\sqrt{3}$ gives $\beta - \alpha = 1$, forcing the middle component of $\vec{v}$ to be -1 with equal first/third components – only $3\hat{i} - \hat{j} + 3\hat{k}$ fits this pattern among the options.

Answer: (C) $3\hat{i} - \hat{j} + 3\hat{k}$

Q.4

$P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$, $Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$. *Relation between* P, Q?

P reduces to $\tan\theta = \sqrt{2} + 1$; Q reduces to $\cot\theta = \sqrt{2} - 1$, i.e. $\tan\theta = 1/(\sqrt{2} - 1) = \sqrt{2} + 1$ as well. Both sets are identical.

Answer: (D) $P = Q$

Q.5

Line $x = b$ divides the area enclosed by $y = (1-x)^2, y = 0, x = 0$ into R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$, with $R_1 - R_2 = \frac{1}{4}$. Find b.

$R_1 = \int_0^b (1-x)^2 dx = \frac{1-(1-b)^3}{3}$; total area is $\frac{1}{3}$, so $R_1 - R_2 = 2R_1 - \frac{1}{3} = \frac{1}{4} \Rightarrow R_1 = \frac{7}{24}$. Solving $1-(1-b)^3 = \frac{7}{8}$ gives $(1-b)^3 = \frac{1}{8} \Rightarrow b = \frac{1}{2}$.

Answer: (B) 1/2

Q.6

α, β roots of $x^2 - 6x - 2 = 0$, $\alpha > \beta$, $a_n = \alpha^n - \beta^n$. Find $\dfrac{a_{10} - 2a_8}{2a_9}$.

Both roots satisfy $t^2 = 6t + 2$, giving the recurrence $a_n = 6a_{n-1} + 2a_{n-2}$, so $a_{10} = 6a_9 + 2a_8$, i.e. $a_{10} - 2a_8 = 6a_9$. The ratio is $6a_9/(2a_9) = 3$.

Answer: (C) 3

Q.7

Line L through $(3, -2)$ inclined at $60°$ to $\sqrt{3}x + y = 1$ (slope $-\sqrt{3}$, angle $120°$), intersecting the x-axis. Equation of L.

L's angle is $120° \pm 60°$; the horizontal choice ($180°$) cannot meet the x-axis from height -2, so L has angle $60°$, slope $\sqrt{3}$. Through $(3, -2)$: $y + 2 = \sqrt{3}(x-3) \Rightarrow y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$.

Answer: (B) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

Q.8

Vector(s) coplanar with $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, perpendicular to $\hat{i} + \hat{j} + \hat{k}$.

Writing $\vec{v} = \lambda(1,1,2) + \mu(1,2,1)$ and imposing $\vec{v} \cdot (1,1,1) = 0$ gives $\lambda = -\mu$, so $\vec{v} = \mu(0,1,-1)$ – any scalar multiple of $\hat{j} - \hat{k}$.

Answer: (A) $\hat{j} - \hat{k}$**, (D)** $-\hat{j} + \hat{k}$

Q.9

M, N non-singular skew-symmetric 3×3 matrices with $MN = NM$. Evaluate $M^2N^2(M^TN)^{-1}(MN^{-1})^T$.

Using $M^T = -M, N^T = -N$ and the commutativity of M, N (hence of their inverses), the expression simplifies step by step to $-M^2$. (This question was officially voided – marks awarded to all candidates – but the algebra points to option (C).)

Answer: Officially voided (marks to all); algebra indicates $-M^2$ (option C)

Q.10

Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has eccentricity reciprocal to that of $x^2 + 4y^2 = 4$, and passes through a focus of the ellipse. Which hold?

The ellipse has $e = \sqrt{3}/2$, so the hyperbola has $e = 2/\sqrt{3}$, giving $b^2/a^2 = 1/3$. Its foci-of-ellipse pass-through point is $(\sqrt{3}, 0)$, giving $a^2 = 3, b^2 = 1$: the hyperbola is $x^2 - 3y^2 = 3$ with $c^2 = a^2 + b^2 = 4$, focus $(2, 0)$.
Answer: (B) a focus is $(2, 0)$, (D) $x^2 - 3y^2 = 3$

Q.11

$f(x + y) = f(x) + f(y)$ for all x, y; f differentiable at $x = 0$. Which hold?

$f(0) = 0$, and for any x, $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{f(h)}{h} = f'(0)$ – a constant. So f is differentiable (hence continuous) everywhere with constant derivative.
Answer: (B), (C) [officially also (D), by vacuous inclusion since f has zero exceptional points]

Q.12

[Paragraph] Real numbers a, b, c satisfy $[a\ b\ c] \begin{pmatrix} 1 & 9 & 7 \\ 8 & 2 & 7 \\ 7 & 3 & 7 \end{pmatrix} = [0\ 0\ 0]$. If $P(a, b, c)$ lies on $2x+y+z = 1$, find $7a + b + c$.

Solving the homogeneous system gives $(a, b, c) = t(-1, -6, 7)$. Substituting into the plane equation: $-2t - 6t + 7t = 1 \Rightarrow t = -1$, so $(a, b, c) = (1, 6, -7)$ and $7a + b + c = 7 + 6 - 7 = 6$.
Answer: (D) 6

Q.13

[Paragraph, same system] ω a root of $x^3 - 1 = 0$ with $\operatorname{Im}\omega > 0$; $a = 2$ with b, c satisfying the system. Find $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c}$.

From $a = -t = 2 \Rightarrow t = -2$: $(b, c) = (12, -14)$. Using $\omega^3 = 1$: $\omega^{12} = 1, \omega^{-14} = \omega$. The sum becomes $3\omega^{-2} + 1 + 3\omega^{-1} = 3\omega + 1 + 3\omega^2 = 1 + 3(\omega + \omega^2) = 1 - 3 = -2$.
Answer: (A) -2

Q.14

[Paragraph, same system] $b = 6$ with a, c satisfying the system; α, β roots of $ax^2 + bx + c = 0$. Find $\sum_{n=0}^{\infty} \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)^n$.

$b = -6t = 6 \Rightarrow t = -1$, giving $(a, c) = (1, -7)$, so $x^2 + 6x - 7 = 0$ with $\alpha + \beta = -6, \alpha\beta = -7$. Then $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{-6}{-7} = \frac{6}{7}$, and the geometric series sums to $\frac{1}{1-6/7} = 7$.
Answer: (B) 7

Q.15

[Paragraph] U_1 (3 white, 2 red), U_2 (1 white). Coin toss: heads $\rightarrow$ draw 1 ball $U_1 \rightarrow U_2$; tails $\rightarrow$ draw 2 balls $U_1 \rightarrow U_2$. Then draw 1 ball from U_2. Probability it is white.

Conditioning on the coin and the colours transferred: $P(\text{white} \mid H) = \frac{4}{5}$ and $P(\text{white} \mid T) = \frac{11}{15}$ (via the hypergeometric draw of 2 balls). Overall, $P(\text{white}) = \frac{1}{2} \cdot \frac{4}{5} + \frac{1}{2} \cdot \frac{11}{15} = \frac{23}{30}$.

Answer: (B) 23/30

Q.16

[Paragraph, same setup] Given the drawn ball from U_2 is white, probability heads appeared.

By Bayes' theorem, $P(H \mid \text{white}) = \dfrac{\frac{1}{2} \cdot \frac{4}{5}}{23/30} = \dfrac{2/5}{23/30} = \dfrac{12}{23}$.

Answer: (D) 12/23