QuestIQ Academy

JEE Advanced 2017 – Paper 1 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = one or more correct; Section 2 = single-digit integer; Section 3 = matching-table, single correct.

Q.1

If $2x-y+1=0$ is tangent to $\dfrac{x^2}{a^2}-\dfrac{y^2}{16}=1$, which CANNOT be sides of a right triangle: (A) $a,4,1$ (B) $a,4,2$ (C) $2a,8,1$ (D) $2a,4,1$?

Tangency condition $c^2=a^2m^2-b^2$ with $m=2,c=1,b^2=16$ gives $a^2=17/4$. Checking each triple against the Pythagorean relation: (A) needs $a^2\in\{15,17,-15\}$ – fails (17/4 matches none). (B) needs $a^2\in\{12,20,-12\}$ – fails. (C) needs $(2a)^2\in\{65,63,-63\}$, i.e. $4a^2=17\neq 65,63$ – fails. (D) needs $(2a)^2=4^2+1^2=17$; since $4a^2=17$ exactly, this WORKS – so (D) can be a right triangle's sides and is correctly excluded.

Answer: (A), (B), (C)

Q.2

A chord (not tangent) of $y^2=16x$ has equation $2x+y=p$ and midpoint (h,k). Which (p,h,k) is possible?

For $y^2=16x$, a chord's slope equals $8/k$ where k is the midpoint's y-coordinate. The given line has slope -2, forcing $k=-4$ – eliminating options (B) and (D) immediately. The midpoint must also satisfy $2h+k=p$: option (A) gives $2(2)-4=0\neq-2$ (fails); option (C) gives $2(3)-4=2=p$ (checks out). Substituting $y=2-2x$ into $y^2=16x$ for option (C) gives $4x^2-24x+4=0$, a real chord with midpoint $x=3,y=-4$, confirming it.

Answer: (C) $p=2,h=3,k=-4$

Q.3

Erratum: Q.39 (about discontinuities of $f(x)=x\cos(\pi(x+[x]))$) contained a printing inconsistency in the original paper. The exam board awarded full marks to all candidates for this question, so no single verified correct option exists.

For reference, using $[x]$ = greatest integer function, $\cos(\pi(x+[x]))=(-1)^{[x]}\cos(\pi x)$, so $f(x)=x(-1)^{[x]}\cos(\pi x)$ on each integer-indexed piece; continuity would need checking separately at each integer using this corrected form.

Answer: Voided by the exam board – full marks awarded to all candidates

Q.4

$f : \mathbb{R} \to (0,1)$ continuous. Which function(s) have a zero in $(0,1)$: (A) $x^9 - f(x)$ (B) $x - \int_0^{\pi/2-x} f(t)\cos t\,dt$ (C) $e^x - \int_0^x f(t)\sin t\,dt$ (D) $f(x) + \int_0^{\pi/2} f(t)\sin t\,dt$?

(A) At $x = 0$: negative (since $-f(0) < 0$); at $x = 1$: positive (since $1 - f(1) > 0$) – IVT gives a zero. True. (B) At $x = 0$: $-\int_0^{\pi/2} f\cos t\,dt \in (-1, 0)$, negative; at $x = 1$: $1 - \int_0^{\pi/2-1} f\cos t\,dt > 1 - \sin(\pi/2 - 1) > 0$ – IVT gives a zero. True. (C) $e^x \geq 1$ while $\int_0^x f(t)\sin t\,dt < \int_0^x \sin t\,dt = 1 - \cos x \leq 1 - \cos 1 \approx 0.46$ throughout $(0,1)$, so this expression stays strictly positive – never zero. False. (D) Both $f(x) > 0$ and the integral (a fixed positive constant) are positive, so the sum is always positive – never zero. False.

Answer: (A), (B)

Q.5

Which matrices are NOT the square of any real 3×3 matrix: (A) I (B) $\mathrm{diag}(1,1,-1)$ (C) $\mathrm{diag}(1,-1,-1)$ (D) $-I$?

For any real matrix M, $\det(M^2) = (\det M)^2 \geq 0$. (A) $\det = 1 \geq 0$ and indeed $I = I^2$ – IS a square. (B) $\det = -1 < 0$ – impossible, NOT a square. (C) $\det = 1 \geq 0$, and using $M = \mathrm{diag}\left(1, \begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ gives $M^2 = \mathrm{diag}(1,-1,-1)$ – IS a square. (D) $\det = -1 < 0$ – impossible, NOT a square.

Answer: (B), (D)

Q.6

$a - b = 1$, $y \neq 0$, $z = x + iy$ satisfies $\mathrm{Im}\left(\dfrac{az+b}{z+1}\right) = y$. Possible values of x?

Direct computation gives $\mathrm{Im}\left(\frac{az+b}{z+1}\right) = \dfrac{y(a-b)}{(x+1)^2 + y^2} = \dfrac{y}{(x+1)^2 + y^2}$ (using $a - b = 1$). Setting this equal to y (and dividing by $y \neq 0$) gives $(x+1)^2 + y^2 = 1 \Rightarrow x = -1 \pm \sqrt{1 - y^2}$.

Answer: (A), (B)

Q.7

$P(X) = 1/3$, $P(X \mid Y) = 1/2$, $P(Y \mid X) = 2/5$.

$P(X \cap Y) = P(Y \mid X)P(X) = \frac{2}{5} \cdot \frac{1}{3} = \frac{2}{15}$. Then $P(Y) = P(X \cap Y)/P(X \mid Y) = \frac{2/15}{1/2} = \frac{4}{15}$ (A true). $P(X' \mid Y) = 1 - P(X \mid Y) = \frac{1}{2}$ (B true). $P(X \cap Y) = 2/15 \neq 1/5$ (C false). $P(X \cup Y) = \frac{1}{3} + \frac{4}{15} - \frac{2}{15} = \frac{7}{15} \neq \frac{2}{5}$ (D false).

Answer: (A), (B)

Q.8

For how many values of p do the circle $x^2 + y^2 + 2x + 4y - p = 0$ and the coordinate axes have exactly three common points?

$p = 0$: the circle passes through the origin, meeting the x-axis again at $(-2, 0)$ and the y-axis again at $(0, -4)$ – exactly 3 distinct points (origin shared). $p = -1$: the circle is tangent to the x-axis (at $(-1, 0)$) and meets the y-axis at 2 distinct points – exactly 3 points. Tangency to the y-axis instead ($p = -4$) leaves no real intersection with the x-axis, giving only 1 point. No other case yields exactly 3.
Answer: 2

Q.9

$f(0) = 0$, $f(\pi/2) = 3$, $f'(0) = 1$. $g(x) = \int_x^{\pi/2}[f'(t)\csc t - \cot t \csc t\, f(t)]\,dt$ on $(0, \pi/2]$. Find $\lim_{x\to 0} g(x)$.

The integrand is exactly $\frac{d}{dt}[f(t)\csc t]$, so $g(x) = [f(t)\csc t]_x^{\pi/2} = f(\pi/2) - f(x)/\sin x = 3 - f(x)/\sin x$. As $x \to 0$, $f(x)/\sin x \to f'(0)\cdot 1 = 1$ (since $f(0) = 0$), so $g(x) \to 3 - 1$.
Answer: 2

Q.10

For real α, the system $\begin{pmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ has infinitely many solutions. Find $1+\alpha+\alpha^2$.

$\det = (1-\alpha^2)^2 = 0 \Rightarrow \alpha = \pm 1$. At $\alpha = 1$ the equations reduce to $x + y + z = 1$ and $x + y + z = -1$ simultaneously – inconsistent. At $\alpha = -1$ all three equations reduce to the single equation $x - y + z = 1$ – infinitely many solutions. So $\alpha = -1$, giving $1 - 1 + 1 = 1$.
Answer: 1

Q.11

Words of length 10 from $\{A, \ldots, J\}$ (10 distinct letters). x = words with no letter repeated; y = words with exactly one letter repeated twice, no other repeats. Find $y/(9x)$.

$x = 10!$ (a permutation of all 10 letters). $y = 10 \cdot \binom{9}{8} \cdot \frac{10!}{2!}$ (choose the repeated letter, choose 8 of the remaining 9 letters, then arrange the 10 symbols). So $y/(9x) = \dfrac{10\cdot 9\cdot 10!/2}{9\cdot 10!} = \dfrac{45\cdot 10!}{9\cdot 10!}$.
Answer: 5

Q.12

Sides of a right triangle are in arithmetic progression, area $= 24$. Find the smallest side.

Writing sides $a - d, a, a + d$ (hypotenuse $a + d$), the right-angle condition gives $a = 4d$, so the sides are $3d, 4d, 5d$ (a scaled 3-4-5 triangle). Area $= \frac{1}{2}(3d)(4d) = 6d^2 = 24 \Rightarrow d = 2$, smallest side $= 3d$.
Answer: 6

Q.13

Matching table: conics, tangent-line formulas, points of contact. For $a = 2$, tangent at $(-1, 1)$.

With $a = \sqrt{2}$, the point $(-1, 1)$ lies on the circle $x^2 + y^2 = a^2 = 2$ (since $1 + 1 = 2$). Its tangent there is $-x + y = 2$, i.e. $y = x + 2$ $(m = 1, c = 2)$. Formula (ii), $y = mx + a\sqrt{m^2 + 1}$, gives $c = \sqrt{2} \cdot \sqrt{2} = 2$ – an exact match. The point-of-contact formula (Q), $\left(\frac{-am}{\sqrt{m^2+1}}, \frac{a}{\sqrt{m^2+1}}\right)$, evaluates to $(-1, 1)$ exactly, confirming circle (I), tangent formula (ii), point (Q).

Answer: (B) (I), (ii), (Q)

Q.14

Tangent $y = x + 8$ has point of contact $(8, 16)$. Identify the matching combination.

This matches the parabola $y^2 = 4ax$ tangent form $my = m^2x + a$: with $m = 1$, $y = x + a$, so $a = 8$ (parabola $y^2 = 32x$). The point-of-contact formula $(a/m^2, 2a/m) = (8, 16)$ matches exactly.

Answer: (C) (III), (i), (P)

Q.15

Tangent to a suitable conic at $(\sqrt{3}, \frac{1}{2})$ is $\sqrt{3}x + 2y = 4$. Identify the combination.

Rewriting as $\frac{\sqrt{3}}{2}x + y = 2$ and comparing to the ellipse $x^2 + a^2y^2 = a^2$'s tangent $\frac{x_1}{a^2}x + y_1y = 1$ at $(\sqrt{3}, \frac{1}{2})$ gives $a^2 = 4$; checking $(\sqrt{3})^2 + 4(\frac{1}{2})^2 = 3 + 1 = 4 = a^2$ confirms the point lies on this ellipse, matching conic (II) with the corresponding formula/point pair.

Answer: (D) (II), (iv), (R)

Q.16

$f(x) = x + \ln x - x \ln x$, $x \in (0, \infty)$. *Matching table about zeros/limits/monotonicity of f, f', f''.*

$f'(x) = \dfrac{1}{x} - \ln x$, $f''(x) = -\dfrac{1}{x^2} - \dfrac{1}{x} < 0$ always (so f' is strictly decreasing everywhere). $f(1) = 1 > 0$, $f(e^2) = 2 - e^2 < 0$ – zero in $(1, e^2)$. $f'(1) = 1 > 0$, $f'(e) = \frac{1}{e} - 1 < 0$ – zero in $(1, e)$. Since f' strictly decreases from $+\infty$ (as $x \to 0^+$) and $f'(1) = 1 > 0$, f' stays positive on $(0, 1)$ – no zero there. f'' is never zero. As $x \to \infty$: $f \to -\infty$, $f' \to -\infty$, $f'' \to 0$.

Answer: (B) (II), (ii), (Q): $f'(x) = 0$ for some $x \in (1, e)$; $\lim_{x\to\infty} f(x) = -\infty$; f is decreasing in (e, e^2)

Q.17

Same setup as Q.52 – second correct combination.

f' is strictly decreasing everywhere (since $f'' < 0$ always), so in particular f' is decreasing throughout (e, e^2), and $\lim_{x\to\infty} f'(x) = -\infty$ as shown above, alongside the zero of f' in $(1, e)$ from Q.52.

Answer: (B) (II), (iii), (S): $f'(x) = 0$ for some $x \in (1, e)$; $\lim_{x\to\infty} f'(x) = -\infty$; f' is decreasing in (e, e^2)

Q.18

Which combination is the only INCORRECT one: (A) (I)(iii)(P) (B) (II)(iv)(Q) (C) (III)(i)(R) (D) (II)(iii)(P)?

Statement (III) claims $f'(x) = 0$ for some $x \in (0, 1)$ – but f' is strictly positive throughout $(0, 1)$ (shown above), so this is false, making any combination built on (III) incorrect.

Answer: (C) (III), (i), (R) – the only INCORRECT combination