QuestIQ Academy

JEE Advanced 2026 – Paper 1 – Mathematics – Worked Solutions

Note: These are QuestIQ Academy's own worked solutions to the Mathematics section only of this paper (Physics and Chemistry are outside QuestIQ Academy's Math-only scope). Section 1 = single correct (Q1–4); Section 2 = one or more correct (Q5–8); Section 3 = numerical value (Q9–12); Section 4 = matching list (Q13–16).

Q.1

$f(x) = \sqrt{x}\ln x - x + 1$ on $(0, \infty)$. Which statement is true?

$f'(x) = \dfrac{\ln x + 2}{2\sqrt{x}} - 1$, and $f''(x) = \dfrac{-\ln x}{4x^{3/2}}$, positive on $(0, 1)$ and negative on $(1, \infty)$: f' increases then decreases, peaking at $x = 1$ where $f'(1) = 0$. So $f'(x) \leq 0$ for all $x > 0$ (equality only at $x = 1$), meaning f is monotonically non-increasing with no sign change in f' – no local extremum exists.

Answer: (D) f has neither a local maximum nor a local minimum in $(0, \infty)$

Q.2

P on $y = x^2$ with tangent slope 4; Q on $x^2 + y^2 = 2$ (Q1) with tangent slope -1; R on $x^2 + 4y^2 = 8$ (Q1) with tangent slope $-\frac{1}{2}$. Radius of circle through P, Q, R.

$P = (2, 4)$, $Q = (1, 1)$, $R = (2, 1)$ (each found by solving the tangent-slope condition). The circumcircle $x^2 + y^2 - 3x - 5y + 6 = 0$ has radius$^2 = \frac{9}{4} + \frac{25}{4} - 6 = \frac{5}{2}$.

Answer: (C) $\sqrt{5/2} = \sqrt{5}/\sqrt{2}$

Q.3

Which matrix is obtainable from I_3 by elementary row transformations?

Elementary row operations preserve invertibility (each is left-multiplication by an invertible elementary matrix), so only a matrix with nonzero determinant can be reached from I. Checking determinants: options (A), (C), (D) are all singular (det = 0); only option (B) has det $= -2 \neq 0$, and every invertible matrix is reachable from I via row reduction (in reverse).

Answer: (B)

Q.4

Evaluate $\cot^{-1}(\cot(-11)) + 10\sin(2\cos^{-1}(1/\sqrt{2})) + 10\sin(2\tan^{-1} 2)$.

$\cot^{-1}(\cot(-11)) = -11+4\pi$ (bringing -11 into $(0,\pi)$ using period π). $2\cos^{-1}(1/\sqrt{2}) = \pi/2 \Rightarrow 10\sin(\pi/2) = 10$. For $\theta = \tan^{-1} 2$: $\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} = \frac{4}{5} \Rightarrow 10\sin 2\theta = 8$. Total: $(4\pi-11)+10+8 = 4\pi+7$.
Answer: (C) $4\pi+7$

Q.5

Box I: 6 red, 9 green; Box II: 8 red, 12 green; mixed together (35 balls total). E_1, E_2: ball from Box I/II; F_1, F_2: ball is red/green.

$P(F_1 \mid E_1) = \frac{6}{15} = \frac{2}{5} = P(F_1) = \frac{14}{35}$: E_1, F_1 independent (A true). $P(F_2 \mid E_2) = \frac{12}{20} = \frac{3}{5} = P(F_2) = \frac{21}{35}$: E_2, F_2 are independent, not dependent (B false). $P(F_1 \mid E_2) = \frac{8}{20} = \frac{2}{5} = P(F_1 \mid E_1)$ (C true). $P(F_1 \mid E_1) = 0.4 < 0.6 = P(F_2 \mid E_2)$ (D false).
Answer: (A), (C)

Q.6

Plane P contains $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{1}$ and is $\perp$ to $x+2y+3z=4$; P_1 through $(4,2,2)$ parallel to P.

Normal of P is perpendicular to both the line's direction $(2,3,1)$ and the given plane's normal $(1,2,3)$: $n_P = (2,3,1)\times(1,2,3) = (7,-5,1)$. Through $(1,3,-2)$: $P: 7x-5y+z=-10$ (A true). Distance from origin $= \frac{10}{\sqrt{75}} = \frac{2\sqrt{3}}{3} \neq 2\sqrt{3}$ (C false). $P_1: 7x-5y+z=20$; distance between $P, P_1 = \frac{30}{\sqrt{75}} = 2\sqrt{3} \neq 30$ (B false). Angle with $2x+2y+z=3$: $\cos\theta = \frac{|14-10+1|}{\sqrt{75}\cdot 3} = \frac{1}{3\sqrt{3}}$ (D true).
Answer: (A), (D)

Q.7

$f:\mathbb{R}\to\mathbb{R}$ arbitrary, $g(x) = xf(x)$.

(A) With $f(x) = 1/x$ $(x\neq 0)$, $f(0)=0$: $g(x)=1$ for $x\neq 0$ but $g(0)=0$ – discontinuous, so "always continuous" is false. (B) $g'(0) = \lim_{h\to 0} f(h) = f(0)$ when f is continuous at 0: true. (C) With $f(x) = \sin(x^2)/x^2$ $(x\neq 0)$, $f(0)=0$ (discontinuous at 0), $g(x) = \sin(x^2)/x$: $g'(0) = \lim \sin(h^2)/h^2 = 1$ exists – g differentiable despite f discontinuous: false. (D) $g'(0) = \lim_{h\to 0}\frac{hf(h)}{h} = \lim_{h\to 0} f(h)$ identically, so $g'(0)$ existing is literally $\lim f(x)$ existing: true.
Answer: (B), (D)

Q.8

$M = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$; $M^{26} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$, $\sum_{k=1}^{26} M^k = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

M has repeated eigenvalue 1 (char. poly $(\lambda-1)^2=0$) and $M\neq I$, so M is similar to the Jordan block $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ (A true). By Cayley–Hamilton, $M^n = nM-(n-1)I$; so $M^{26} = 26M-25I = \begin{pmatrix} 27 & -26 \\ 26 & -25 \end{pmatrix}$. Summing: $\sum_{k=1}^{26} M^k = 351M-325I = \begin{pmatrix} 377 & -351 \\ 351 & -325 \end{pmatrix}$, so $a = 377 \neq 378$ (B false). Since $\det M = 1$, $\det M^{26} = 1$, giving integer inverse and a unique integer solution for any integer

(m,n) (C true). For (D): $\det\begin{pmatrix} a+t & b \\ c & d+t \end{pmatrix} = t^2+52t+676 = (t+26)^2 > 0$ for all $t>0$: unique solution always exists (D true).
Answer: (A), (C), (D)

Q.9

$S=\{1,\dots,10\}$. $|X|$ where X = equivalence relations on S with exactly 42 ordered pairs.

An equivalence relation corresponds to a partition into blocks of sizes n_i with $\sum n_i = 10$ and $\sum n_i^2 = 42$ (pairs within each block). Checking partitions: $\{1,4,5\}$ gives $1+16+25=42$, and $\{1,1,2,6\}$ gives $1+1+4+36=42$ (no other partition shape works). Counting labelled partitions: shape $\{1,4,5\}$: $\binom{10}{1}\binom{9}{4}\binom{5}{5} = 1260$. Shape $\{1,1,2,6\}$ (two indistinguishable singletons): $\frac{\binom{10}{1}\binom{9}{1}}{2!}\binom{8}{2}\binom{6}{6} = 45\times 28 = 1260$. Total $= 1260+1260 = 2520$.
Answer: 2520

Q.10

$f(x) = (|x|+|x-1|)\sin x + \lfloor x\sin x\rfloor$ on $(-\frac{\pi}{2},\frac{\pi}{2})$. α = discontinuity count, β = non-differentiability count. Find $\alpha+\beta$.

$(|x|+|x-1|)\sin x$ is continuous everywhere; its corner at $x=0$ is smoothed by $\sin 0 = 0$ (matching one-sided derivatives $=1$), but the corner at $x=1$ survives since $\sin 1 \neq 0$ – one extra non-differentiable point. The floor term $\lfloor x\sin x\rfloor$ jumps wherever $x\sin x$ crosses an integer; since $x\sin x$ is even, ranges over $[0,\pi/2)\approx[0,1.57)$, and crosses only the integer 1 (at two symmetric points), $\alpha=2$. These 2 points are also non-differentiable, plus the corner at $x=1$: $\beta=3$. So $\alpha+\beta=5$.
Answer: 5

Q.11

Distribute 10 identical red pens and 14 identical blue pens among 4 persons so each gets exactly 6 pens total.

With r_i red pens for person i ($0\le r_i\le 6$, since blue $=6-r_i\ge 0$) and $\sum r_i = 10$: unconstrained count $\binom{13}{3}=286$; subtract cases with some $r_i\ge 7$: $4\binom{6}{3}=80$ (no double-violations possible since that would need sum $\ge 14>10$). Total $=286-80=206$.
Answer: 206

Q.12

$\alpha = \prod(1-2\cos(k\pi/11))$ for $k=1,3,9,27,81$ (angles mod 2π, reduced to the odd multiples $1,3,5,7,9$ of $\pi/11$). Find $5-\alpha^2$.

Reducing $27\pi/11 \to 5\pi/11$ and $81\pi/11 \to 7\pi/11$ (mod 2π, using evenness of cosine), the product runs over all five odd multiples of $\pi/11$ from 1 to 9. This classical product evaluates to $\alpha=1$ exactly.
Answer: 4

Q.13

[Matching] α, β roots of $x^2+x+1=0$ (ω, ω^2); match reciprocal-power expressions to their resulting equations/values.

(P) $1/(\alpha+1)^{2026} = 1/(-\omega^2)^{2026} = \omega$, $1/(\beta+1)^{2026} = \omega^2$: these are roots of $x^2+x+1=0$: (P)→(1). (Q) Similarly $1/(\alpha+1)^{2027} = -\omega^2$, $1/(\beta+1)^{2027} = -\omega$: sum = 1, product = 1, giving $x^2-x+1=0$: (Q)→(2). (R) With γ, δ roots of $x^2-x+1=0$ (i.e. $e^{\pm i\pi/3}$), $\gamma-1=\omega, \delta-1=\omega^2$, giving $1/\omega^{2026}+1/\omega^{2\cdot 2026} = \omega^2+\omega = -1$: (R)→(4). (S) With $p+r=-1, pr=-1$ for $x^2+x-1=0$: setting $u=p+1, v=r+1$ $(u+v=1, uv=-1)$, $1/u^3+1/v^3 = (u^3+v^3)/(uv)^3 = 4/(-1) = -4$: (S)→(5).

Answer: (C) P→1, Q→2, R→4, S→5

Q.14

[Matching] Count solutions of four trigonometric equations on given intervals.

(P) $\sin^6 x + \cos^4 x = 1$ on $[-\pi, \pi]$: with $s = \sin^2 x$, $s(s+2)(s-1) = 0 \Rightarrow s = 0, 1$, giving $x = 0, \pm\pi, \pm\pi/2$: 5 points: (P)→(5). (Q) $\sin^2 x + \cos^6 x = 1$ on $[-\pi/2, \pi/2]$: $-s(s-1)(s-2) = 0 \Rightarrow s = 0, 1$, giving $x = 0, \pm\pi/2$: 3 points: (Q)→(3). (R) $\cos^2(x/2) - \sin^2 x = 1/2$ on $[-\pi, \pi]$ reduces to $2c^2+c-2=0$ $(c = \cos x)$, one valid root $c \approx 0.781$, giving $x = \pm\cos^{-1}(0.781)$: 2 points: (R)→(2). (S) $6\sin^2(x/2) - \cos 3x = 3$ on $[-2\pi, 2\pi]$ reduces to $\cos x = 0$, giving $x = \pm\pi/2, \pm 3\pi/2$: 4 points: (S)→(4).

Answer: (B) P→5, Q→3, R→2, S→4

Q.15

[Matching] Orthogonal matrix M with rows as given; $\vec{u}, \vec{v}, \vec{w}$ built from its entries.

Row-orthonormality forces $\alpha = 0$, $\beta = 1/\sqrt{3}$, and the third row (via cross product of the first two) $= (2/\sqrt{6}, -1/\sqrt{6}, -1/\sqrt{6})$. (P) $\gamma^2+\delta^2 = \frac{4}{6}+\frac{1}{6} = \frac{5}{6}$: (P)→(5). (Q) Solving $x\vec{u}+y\vec{v}+z\vec{w} = \hat{j}$ gives $x = y = z = 1/\sqrt{3}$: (Q)→(4). (R) $\vec{u}\cdot(\vec{v}\times\vec{w}) = 1$ (direct computation): (R)→(2). (S) Since $\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w} = 0$, the vector triple product $\vec{u}\times(\vec{v}\times\vec{w}) = \vec{v}(\vec{u}\cdot\vec{w}) - \vec{w}(\vec{u}\cdot\vec{v}) = \vec{0}$: (S)→(1).

Answer: (A) P→5, Q→4, R→2, S→1

Q.16

[Matching] Four geometry problems: circle, common tangent, ellipse normal, hyperbola.

(P) Circle centre $(1,2)$ tangent to $3x+4y=1$: radius $= \frac{|3+8-1|}{5} = 2$; circle $(x-1)^2+(y-2)^2 = 4$ passes through $(3,2)$: (P)→(3). (Q) Common tangent (positive slope) to $x^2+y^2=2$ and $y^2=8x$: solving $c^2 = 2(1+m^2)$, $c = 2/m$ gives $m=1, c=2$; line $y = x+2$ passes through $(7,9)$: (Q)→(2). (R) Ellipse $\frac{x^2}{16}+\frac{y^2}{12} = 1$, latus-rectum point $M = (2,3)$; normal $2x-y=1$ passes through $(1,1)$: (R)→(1). (S) Hyperbola with $c=5$, directrix magnitude $16/5 = a^2/c \Rightarrow a = 4$, $b^2 = 9$: $\frac{x^2}{16}-\frac{y^2}{9} = 1$ passes through $(8, 3\sqrt{3})$: (S)→(5).

Answer: (B) P→3, Q→2, R→1, S→5