Total Lecture- 5

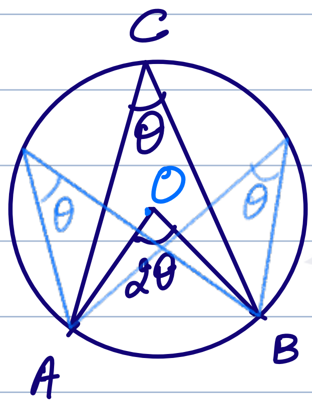
Circle properties.

- Arc property
- Cyclic property.
- Tangent property.
- Chord property.

Lecture-I

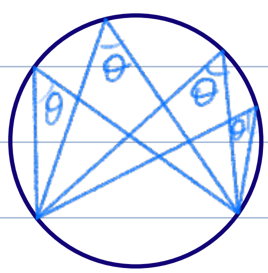
Theorem-1

The angle substended by an arc of a Circle at the Centre is double the angle substended by it at any point on the remaining part of the circle.



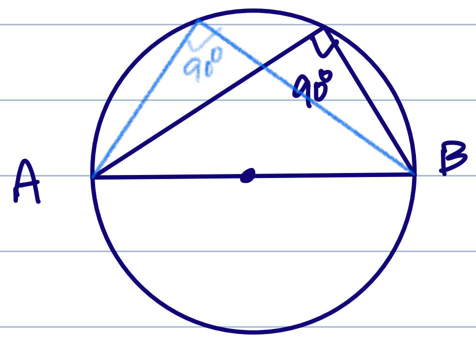
Theorem-2

Angle in the same segment of a circle are equal.

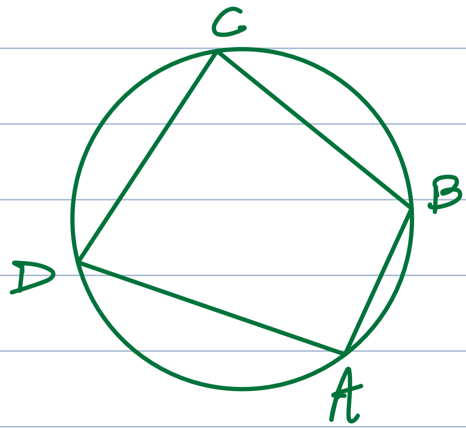


Theorem-3

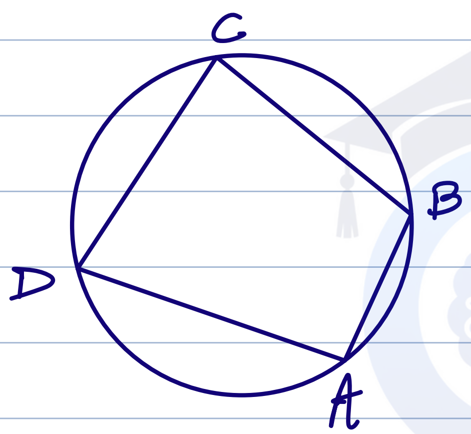
The angle in a Semi-circle is a Right angle.



Cyclic quadrilateral → Quadrilateral whose all the vertices lies on the circumference of a circle.

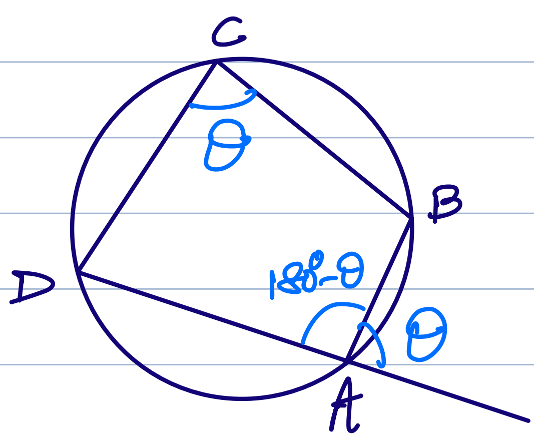


Theorem-4 The sum of opposite angles of a cyclic quadrilateral is 180°.

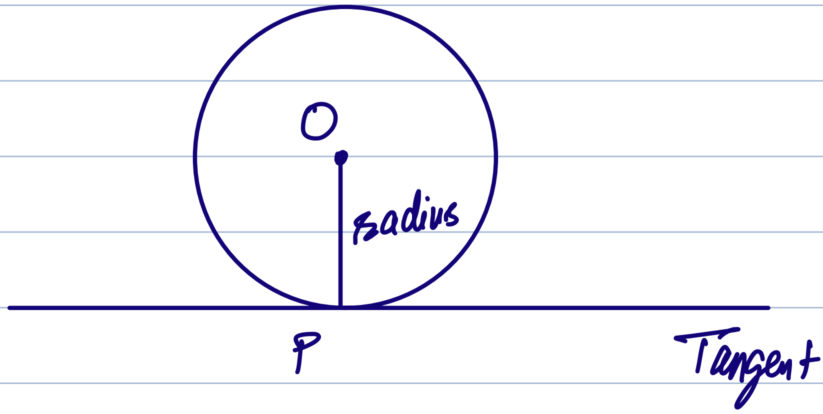


$$\angle A + \angle C = 180^\circ$$
$$\angle B + \angle D = 180^\circ$$

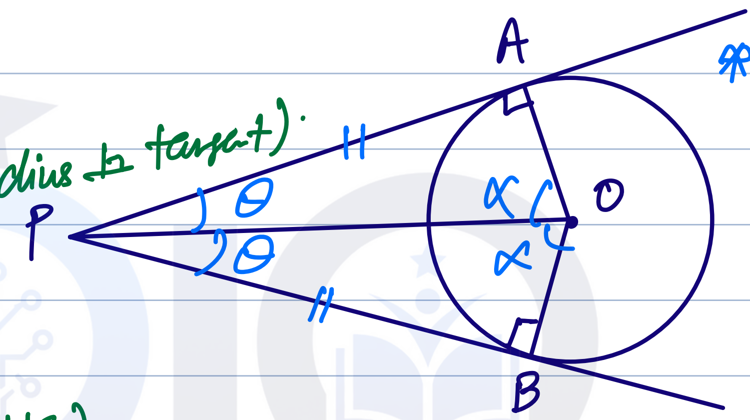
Theorem-5 Exterior angle of a cyclic quadrilateral is equal to opposite interior angle.



Theorem-6 Radius is always perpendicular to the tangent.



Theorem-7



$\circledast$ $\alpha + \theta = 90^\circ$.

In $\triangle OAP$ & $\triangle OBP$

$\angle OAP = \angle OBP$ ($= 90^\circ$, $\because$ radius $\perp$ to tangent).

$OA = OB$ ($=$ radii)

$OP = OP$ (Common)

$\Rightarrow \triangle OAP \cong \triangle OBP$ (by RHS)

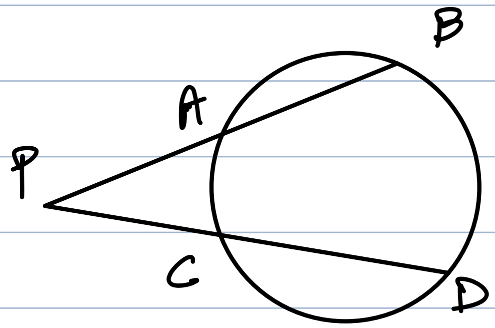
$\Rightarrow$ $\boxed{PA = PB}$ $\rightarrow$ Tangents drawn from an external point to a circle are of equal lengths.

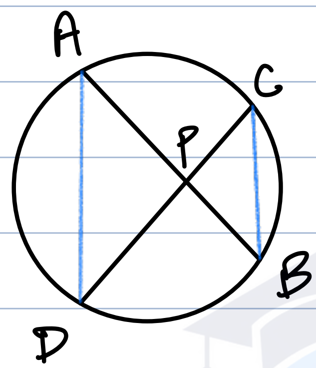
$\Rightarrow$ $\boxed{\angle AOP = \angle BOP}$ $\rightarrow$ Tangents subtends equal angles at the centre.

$\Rightarrow$ $\boxed{\angle APO = \angle BPO}$ $\rightarrow$ Tangents are equally inclined to the line joining the point & the centre of the circle.

Theorem - 8

If two chords of a circle intersects externally or internally, the the products of length of their segments are equal.

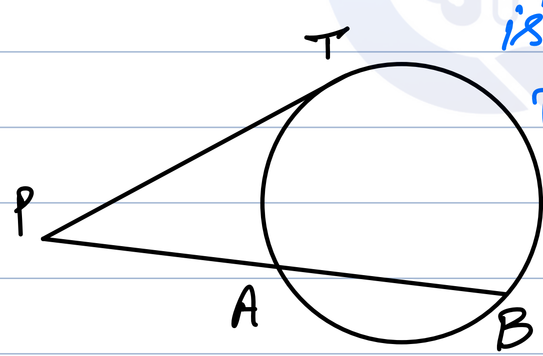




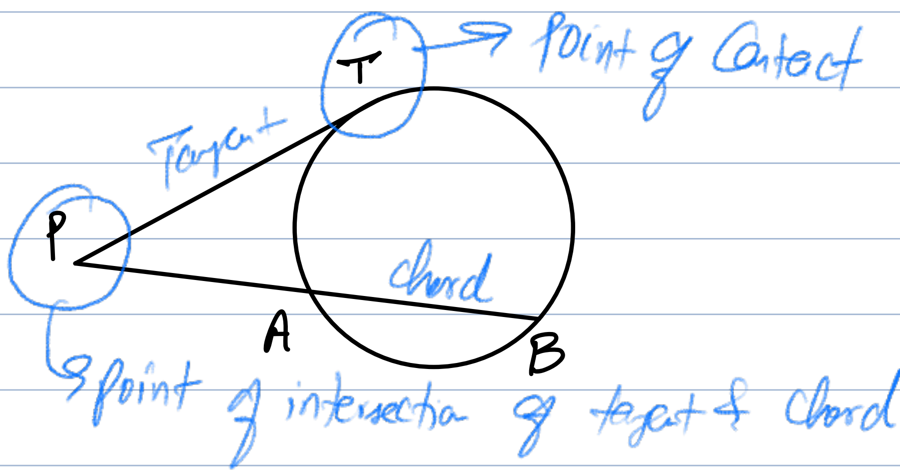
$$\boxed{PA \times PB = PC \times PD}$$

Theorem - 9

If a chord & a tangent intersect externally, then the product of the lengths of segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

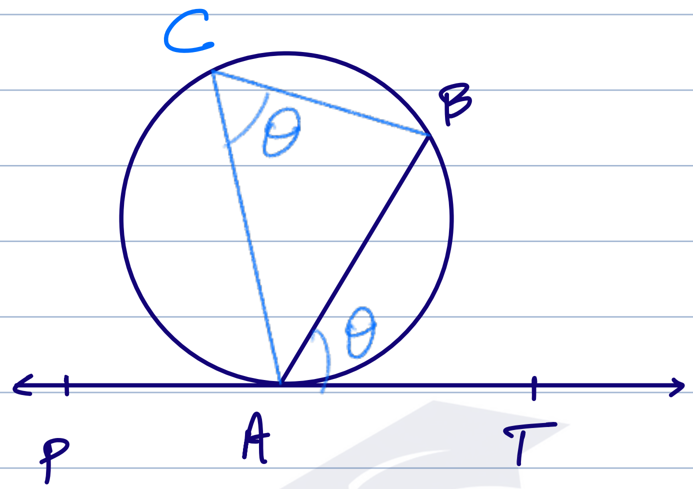


$$\boxed{PA \times PB = (PT)^2}$$

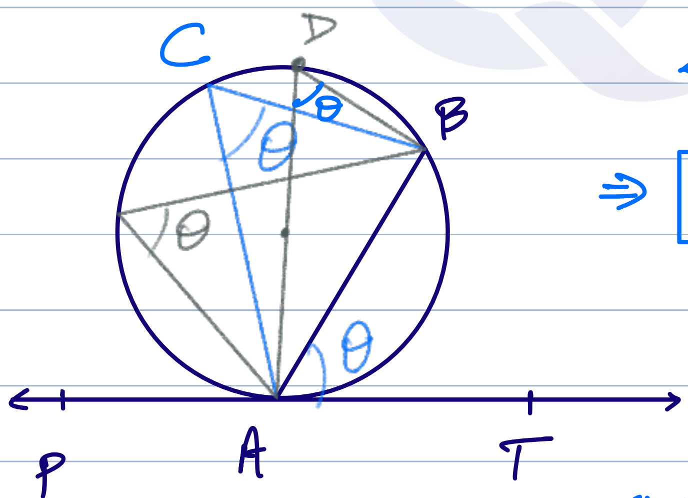


Theorem-10 (Alternate Segment Theorem)

The angle between a tangent & a chord through the point of contact is equal to an angle in alternate segment.



Pf:-



$\angle ABD = 90^\circ$ (Angle in a semi-circle)

$\Rightarrow \boxed{\angle DAB + \angle BDA = 90^\circ} \rightarrow (1)$

Radius ⊥ T.

$\boxed{\angle DAB + \angle BAT = 90^\circ} \rightarrow (2)$

from (1) & (2)

$\cancel{\angle DAB} + \angle BDA = \cancel{\angle DAB} + \angle BAT$

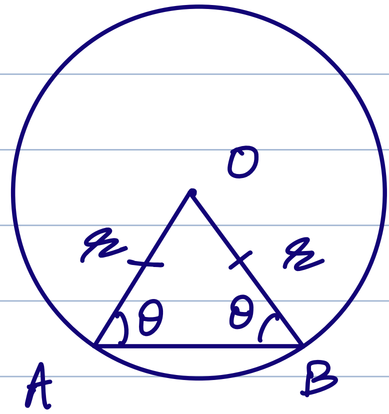
$\Rightarrow \angle BDA = \angle BAT$

$\Rightarrow \angle BCA = \angle BAT$

[$\because \angle BDA = \angle BCA$; Angle on the same segment]

Lecture-II

Recall

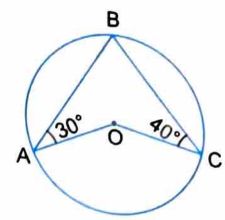


Exercise-18

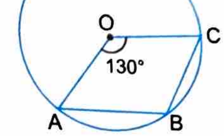
* Class-work → "✓" Marked questions.
* Home-work → " " Bullet point questions.

EXERCISE 18

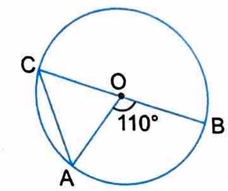
1. *In the given figure, O is the centre of a circle,* $\angle OAB = 30°$ *and* $\angle OCB = 40°$*. Calculate* $\angle AOC$*.*
[**Hint.** *Join OB.*]



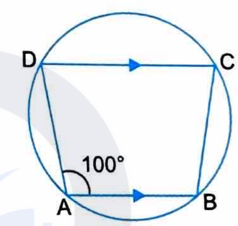
2. In the given figure, O is the centre of the circle and $\angle AOC = 130°$. Find $\angle ABC$.
[**Hint.** *Reflex* $\angle AOC = (360° - 130°) = 230°$]



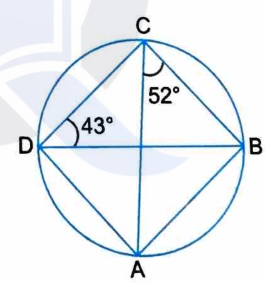
3. In the given figure, O is the centre of the circle and $\angle AOB = 110°$. Calculate : (*i*) $\angle ACO$ (*ii*) $\angle CAO$



4. In the given figure, AB || DC and $\angle BAD = 100°$. Calculate : (*i*) $\angle BCD$ (*ii*) $\angle ADC$ (*iii*) $\angle ABC$
[**Hint.** *ABCD is a cyclic quadrilateral.*]



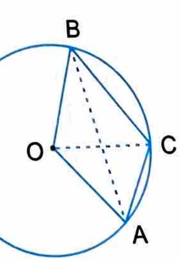
5. In the given figure, $\angle ACB = 52°$ and $\angle BDC = 43°$. Calculate : (*i*) $\angle ADB$ (*ii*) $\angle BAC$ (*iii*) $\angle ABC$.
[**Hint.** *Look at the angles in the same segment.*]



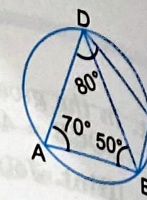
6. In the given figure, O is the centre of the circle. If $\angle AOB = 140°$ and $\angle OAC = 50°$, find :
(*i*) $\angle ABC$ (*ii*) $\angle BCO$ (*iii*) $\angle OAB$ (*iv*) $\angle BCA$.
[**Hint.** (*i*) $OA = OC \Rightarrow \angle OCA = \angle OAC = 50°$. *Find* $\angle AOC$.

$$\angle ABC = \frac{1}{2} \angle AOC.$$

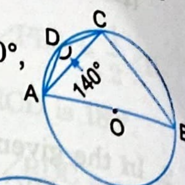
(*ii*) $\angle BOC = (140° - 80°) = 60°$
$\angle OBC + \angle OCB + \angle BOC = 180° \Rightarrow x + x + 60° = 180°$. *Find x.*
(*iii*) $\angle OAB + \angle OBA + \angle AOB = 180° \Rightarrow y + y + 140° = 180°$. *Find y.*
(*iv*) $\angle BAC = \frac{1}{2} \angle BOC = 30°$ *and* $\angle ABC + \angle BAC + \angle BCA = 180°$]



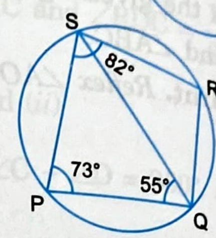
7. In the given figure, ∠BAD = 70°, ∠ABD = 50° and ∠ADC = 80°.
Calculate (*i*) ∠BDC (*ii*) ∠BCD (*iii*) ∠BCA
[**Hint :** *∠BCA = ∠ADB* (*∠s in the same segment*)]



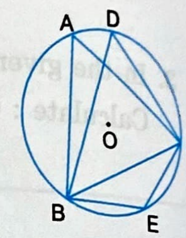
8. (*i*) In the given figure, O is the centre of the circle. If ∠ADC = 140°, find ∠BAC.
[**Hint :** *∠ABC = 40° and ∠ACB = 90°.*]



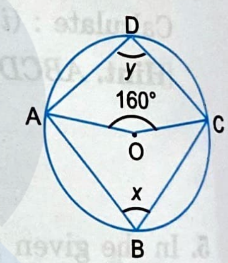
(*ii*) PQRS is a cyclic quadrilateral.
Given ∠QPS = 73°, ∠PQS = 55° and ∠PSR = 82°, calculate ∠QRS, ∠RQS and ∠PRQ. **(2018)**



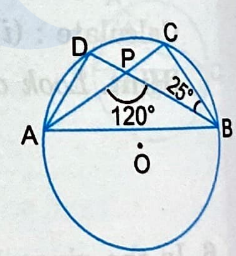
9. In the given figure, O is the centre of the circle and ΔABC is equilateral.
Find : (*i*) ∠BDC and (*ii*) ∠BEC.



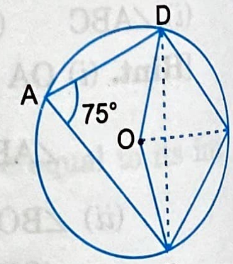
10. (*i*) In the given figure, O is the centre of the circle and ∠AOC = 160°.
Prove that 3∠*y* – 2∠*x* = 140°. **(2005)**



(*ii*) In the given figure, O is the centre of the circle.
If ∠CBD = 25° and ∠APB = 120°, find ∠ADB.
[**Hint.** *∠CPB* = (180° – 120°) = 60°.]



11. In the given figure, O is the centre of the circle, ∠BAD = 75° and chord BC = chord CD.
Find : (*i*) ∠BOC (*ii*) ∠OBD (*iii*) ∠BCD **(2009)**
[**Hint :** *∠BAD + ∠BCD* = 180° ⇒ *∠BCD* = 105°
∠BOD = ∠2BAD = (2 × 75) = 150°.
OB = OD ⇒ ∠ODB = ∠OBD = x°
⇒ *x + x* + 150 = 180 ⇒ 2*x* = 30 ⇒ *x* = 15
∠CBD = ∠CDB = y°
Now, y + y + 105 = 180° ⇒ 2*y* = 75 ⇒ *y* = 37.5
∠OBC = (15° + 37.5°) = 52.5°
∠BOC = 180° – (*∠OBC + ∠OCB*) = (180° – 105°) = 75°]

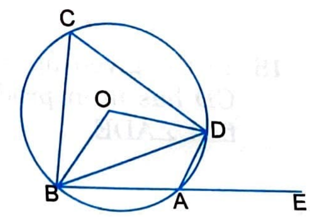


12. In the figure given, O is the centre of the circle. $\angle DAE = 70°$, find giving suitable reasons, the measure of: **(2017)**

(i) $\angle BCD$

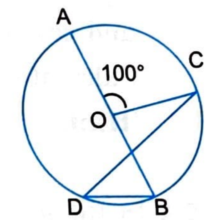
(ii) $\angle BOD$

(iii) $\angle OBD$



13. In the given figure, AOB is a diameter of the circle with centre O and $\angle AOC = 100°$, find $\angle BDC$.

[**Hint.** $\angle BOC = 80°$.]



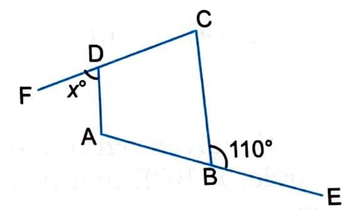
14. In the given figure, find whether the points A, B, C, D are concyclic when :

(i) $x = 70$ (ii) $x = 80$

[**Hint :** $\angle ADC = 180° - x$ *and* $\angle ABC = 70°$

(i) *When* $x = 70$, $\angle ADC + \angle ABC = 180°$

(ii) *When* $x \neq 70$, $\angle ADC + \angle ABC \neq 180°$.]



15. In the adjoining figure, $\angle BAD = 65°$, $\angle ABD = 70°$ and $\angle BDC = 45°$.

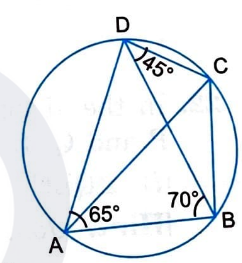
Find : (i) $\angle BCD$ (ii) $\angle ADB$

Hence, show that AC is a diameter. **(2006)**

[**Hint.** (i) *Since ABCD is cyclic, we have* $\angle BAD + \angle BCD = 180°$

(ii) *In* ΔABD, $\angle BAD + \angle ABD + \angle ADB = 180° \Rightarrow \angle ADB = 45°$

$\therefore \angle ADC = \angle ADB + \angle BDC = 90°$ *and hence AC is a diameter.*]



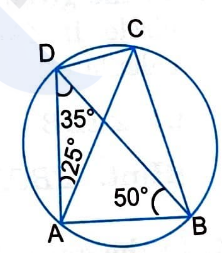
16. In the given figure, ABCD is a cyclic quadrilateral in which $\angle CAD = 25°$, $\angle ADB = 35°$ and $\angle ABD = 50°$.

Calculate : (i) $\angle CBD$ (ii) $\angle CAB$ (iii) $\angle ACB$

[**Hint.** (ii) $\angle ACD = \angle ABD = 50°$

$\therefore \angle BDC = 180° - (25° + 50°) = 105°$

$\therefore \angle CAB = \angle BDC = 105°$]



17. In the figure, AB is parallel to DC, $\angle BCE = 80°$ and $\angle BAC = 25°$. **(2008)**

Find : (i) $\angle CAD$ (ii) $\angle CBD$ (iii) $\angle ADC$

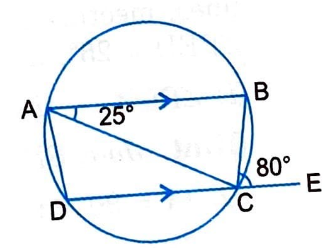
Hint. (i) *Since the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle, we have*

$\angle BAD = \angle BCE \Rightarrow 25° + \angle CAD = 80° \Rightarrow \angle CAD = 55°$

(ii) $\angle CBD = \angle CAD = 55°$ [$\angle s$ *in the same segment*]

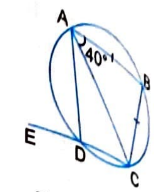
(iii) $\angle ABC + \angle ADC = 180° \Rightarrow \angle BCE + \angle ADC = 180° \Rightarrow \angle ADC = 100°$.

[$\therefore \angle ABC = \angle BCE = 80°$ (*alt.* $\angle s$)]



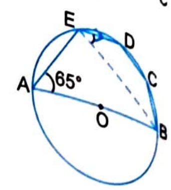
Lecture-III

18. In the given figure, ABCD is a cyclic quadrilateral whose side CD has been produced to E. If BA = BC and $\angle BAC = 40°$, find $\angle ADE$.



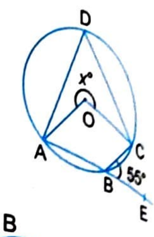
19. In the given figure, AB is a diameter of a circle with centre O and chord ED is parallel to AB and $\angle EAB = 65°$. Calculate : (i) $\angle EBA$ (ii) $\angle BED$ (iii) $\angle BCD$

[**Hint.** *Join EB*]



20. In the given figure, O is the centre of a circle and ABE is a straight line. If $\angle CBE = 55°$, find :

(i) $\angle ADC$ (ii) $\angle ABC$ (iii) the value of x.

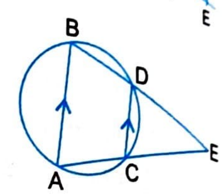


21. In the given figure AB and CD are two parallel chords of a circle. If BDE and ACE are straight lines, intersecting at E, prove that Δ AEB is isosceles.

[**Hint.** $\angle EDC = \angle A$, $\angle DCE = \angle B$.

But, $AB \parallel CD \Rightarrow \angle EDC = \angle B$ *and* $\angle DCE = \angle A$.

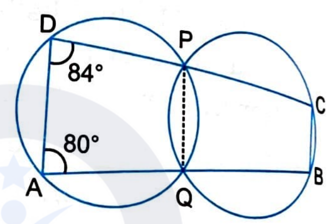
$\therefore \angle A = \angle B$.]



22. In the given figure, the two circles intersect at P and Q. If $\angle A = 80°$ and $\angle D = 84°$, calculate :

(i) $\angle QBC$ (ii) $\angle BCP$.

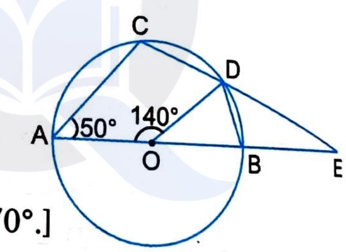
[**Hint.** *Join PQ.*]



23. In the given figure, O is the centre of the circle. If $\angle AOD = 140°$ and $\angle CAB = 50°$, calculate :

(i) $\angle EDB$ (ii) $\angle EBD$

[**Hint.** $\angle BOD = 40°$ *and* $OB = OD \Rightarrow \angle OBD = \angle ODB = 70°$.]



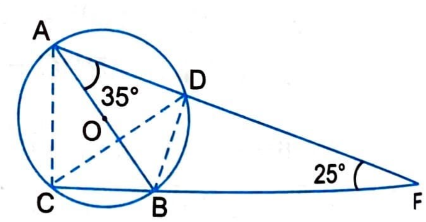
24. In the given figure, AB is a diameter of a circle with centre O. If ADE and CBE are straight lines, meeting at E such that $\angle BAD = 35°$ and $\angle BED = 25°$, find :

(i) $\angle DCB$ (ii) $\angle DBC$ (iii) $\angle BDC$.

[**Hint.** *Join BD, CA and CD.*

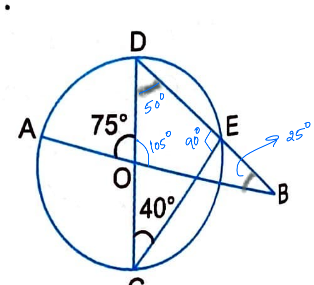
$\angle BDA = 90° \Rightarrow \angle EDB = 90°$. $\angle EBD = 180° - (90° + 25°) = 65°$.

$\therefore \angle DBC = (180° - 65°) = 115°$. *Also,* $\angle DCB = \angle BAD = 35°$.]



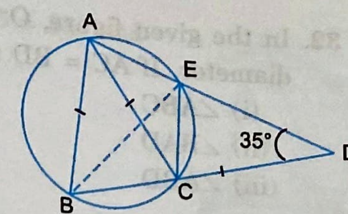
25. In the given figure, the straight lines AB and CD pass through the centre O of the circle. If $\angle AOD = 75°$ and $\angle OCE = 40°$, find :

(i) $\angle CDE$ (ii) $\angle OBE$.



26. In the adjoining figure, AB = AC = CD and $\angle ADC = 35°$. Calculate : (*i*) $\angle ABC$ (*ii*) $\angle BEC$

[**Hint.** $AC = CD \Rightarrow \angle CAD = \angle ADC = 35°$.
$\angle ACB = (35° + 35°)$ *and*
$\angle ABC = \angle ACB = 70°$.
$\therefore \angle BAC = 40° = \angle BEC$.]



27. The exterior angles B and C in ΔABC are bisected to meet at a point P. Prove that $\angle BPC = 90° - \frac{A}{2}$. Is ABPC a cyclic quadrilateral?

[**Hint.** $\angle CBD = (\angle A + \angle C)$ *and* $\angle BCE = (\angle A + \angle B)$

$$\frac{1}{2}\angle CBD + \frac{1}{2}\angle BCE + \angle BPC = 180°$$

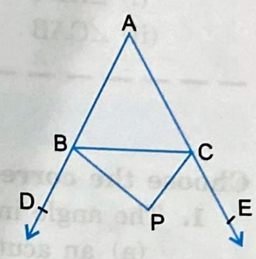
$$\Rightarrow \frac{1}{2}(\angle A + \angle C) + \frac{1}{2}(\angle A + \angle B) + \angle BPC = 180°$$

$$\Rightarrow \frac{1}{2}(\angle B + \angle C) + \angle BPC = 180° - A$$

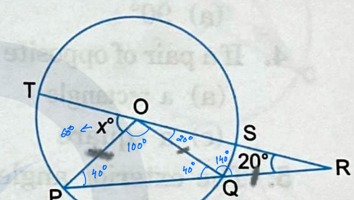
$$\Rightarrow \frac{1}{2}(180° - A) + \angle BPC = 180° - A \Rightarrow \angle BPC = \left(90° - \frac{A}{2}\right)$$

$$\therefore \angle A + \angle BPC = \left(90° - \frac{A}{2}\right) + A = \left(90° + \frac{A}{2}\right) \neq 180°$$

Hence ABPC is not cyclic.]

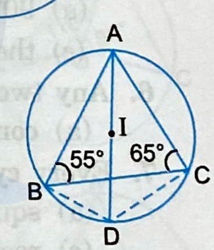


28. In the given figure, O is the centre of the circle. If QR = OP and $\angle ORP = 20°$, find the value of x giving reasons. **(2018)**



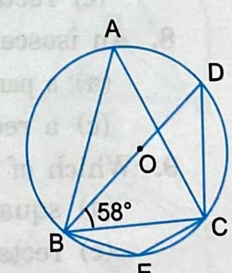
29. In the given figure, I is the incentre of ΔABC. Here AI produced meets the circumcircle of ΔABC at D. If $\angle ABC = 55°$ and $\angle ACB = 65°$, calculate :

(*i*) $\angle BCD$ (*ii*) $\angle CBD$ (*iii*) $\angle DCI$ (*iv*) $\angle BIC$



30. In the given figure, $\angle DBC = 58°$ and BD is a diameter of the circle. Calculate : (*i*) $\angle BDC$ (*ii*) $\angle BEC$ (*iii*) $\angle BAC$ **(2014)**

[**Hint :** (*i*) *Since BD is a diameter, so* $\angle BCD = 90°$.
In ΔDBC, $\angle BDC = 180° - (90° + 58°) = 32°$.
(*ii*) *From cyclic quad. ABEC, we have*
$\angle BEC = (180° - 32°) = 148°$.
(*iii*) $\angle BAC = \angle BDC = 32°$. (*Angles in the same segment*)]



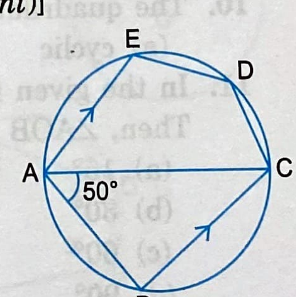
31. In the given figure, ABCDE is a pentagon inscribed in a circle such that AC is a diameter and side BC||AE. If $\angle BAC = 50°$, find giving reasons :

(*i*) $\angle ACB$

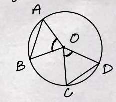
(*ii*) $\angle EDC$

(*iii*) $\angle BEC$

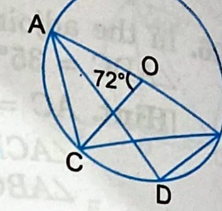
Hence prove that BE is also a diameter. **(2019)**



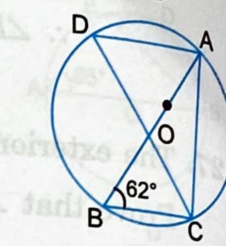
Note: Equal chords subtends equal angle at the centre of a circle



32. In the given figure, O is the centre of the circle and AB is a diameter. If AC = BD and $\angle AOC = 72°$, find
(i) $\angle ABC$
(ii) $\angle BAD$
(iii) $\angle ABD$ **(2020)**



33. In the given figure, A, C, B, D are points on the circle with centre O. Given $\angle ABC = 62°$. Find.
(i) $\angle ADC$
(ii) $\angle CAB$ **(2022, Semester 2)**



MULTIPLE CHOICE QUESTIONS

Choose the correct option :

1. The angle in a semi-circle is :
(a) an acute angle (b) an obtuse angle
(c) a right angle (d) can be either acute or obtuse

2. The angle subtended by an arc of a circle at any point on the remaining part of the circle is ________ of the angle subtended by it at the centre.
(a) double (b) half (c) equal (d) triple

3. The sum of the opposite angles of a cyclic quadrilateral is :
(a) 90° (b) 150° (c) 180° (d) 360°

4. If a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is :
(a) a rectangle (b) a parallelogram
(c) a square (d) a cyclic quadrilateral

5. The exterior angle of a cyclic quadrilateral is equal to :
(a) 90° (b) the interior opposite angle
(c) the interior adjacent angle (d) any of the interior angles

6. Any two angles formed in the same segment of as circle are :
(a) complementary (b) supplementary (c) each right angle (d) equal

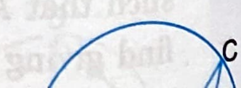
7. Every cyclic parallelogram is a/an :
(a) square (b) rhombus
(c) rectangle (d) isosceles trapezium

8. An isosceles trapezium is always :
(a) a parallelogram (b) a square
(c) a rectangle (d) a cyclic quadrilateral

9. Which of the following quadrilaterals is not always a cyclic quadrilateral?
(a) square (b) rhombus
(c) rectangle (d) an isosceles trapezium

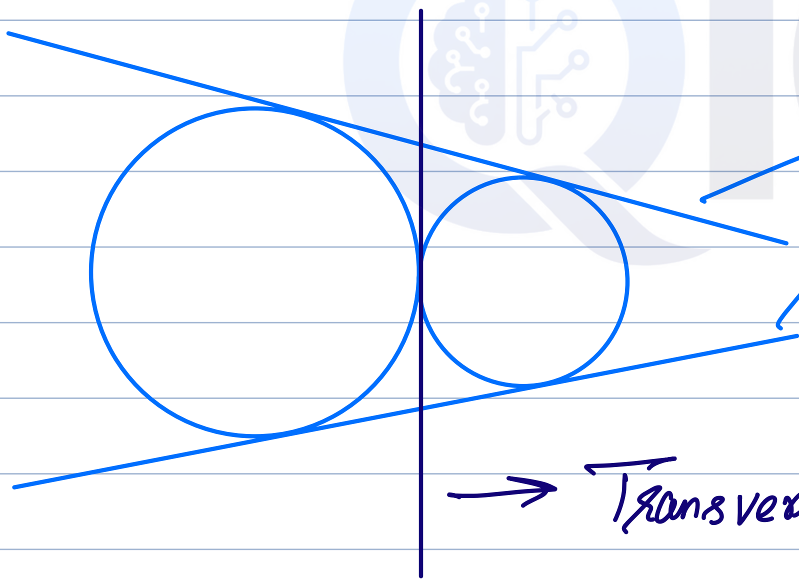
10. The quadrilateral formed by angle bisectors of a cyclic quadrilateral is :
(a) cyclic (b) square (c) rectangle (d) parallelogram

11. In the given figure, O is the centre of the circle and $\angle ACB = 30°$. Then, $\angle AOB = ?$
(a) 15°
(b) 30°



Lecture - IV

Note:-

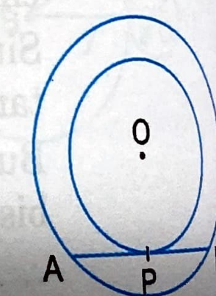


Exercise- 19A.

* Class-work $\longrightarrow$ "✓" Marked questions.
* Home-work $\longrightarrow$ "●" Bullet point questions.

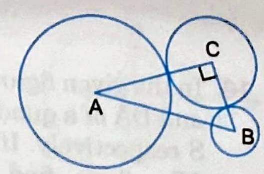
EXERCISE 19A

1. Find the length of the tangent drawn to a circle of radius 8 cm, from a point which is at a distance of 10 cm from the centre of the circle.
2. A point P is 17 cm away from the centre of the circle and the length of tangent drawn from P to the circle is 15 cm. Find the radius of the circle.
3. There are two concentric circles, each with centre O and of radii 10 cm and 26 cm respectively. Find the length of the chord AB of the outer circle which touches the inner circle at P.



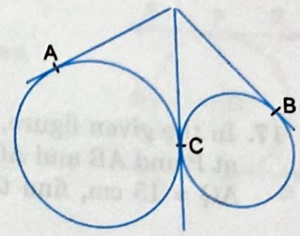
4. A and B are centres of circles of radii 9 cm and 2 cm such that AB = 17 cm and C is the centre of the circle of radius r cm which touches the above circles externally. If $\angle ACB = 90°$, write an equation in r and solve it. **[2004]**

[**Hint :** $AB^2 = AC^2 + BC^2 \Rightarrow (17)^2 = (9 + r)^2 + (2 + r)^2$. *Find r.*]



5. Two circles touch each other externally at a point C and P is a point on the common tangent at C. If PA and PB are tangents to the two circles, prove that PA = PB.

[**Hint :** *PA = PC and PB = PC (Why?).*]



6. Two circles touch each other internally. Prove that the tangents drawn to the two circles from any point on the common tangent are equal in length.

7. Two circles of radii 18 cm and 8 cm touch externally. Find the length of a direct common tangent to the two circles.

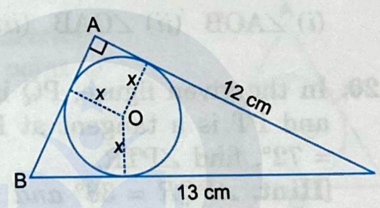
8. Two circles of radii 8 cm and 3 cm have their centres 13 cm apart. Find the length of a direct common tangent to the two circles.

9. Two circles of radii 8 cm and 3 cm have a direct common tangent of length 10 cm. Find the distance between their centres, upto two places of decimal.

10. With the vertices of ΔPQR as centres, three circles are described, each touching the other two externally. If the sides of the triangle are 7 cm, 8 cm and 11 cm, find the radii of the three circles.

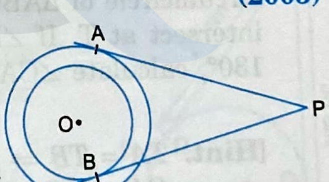
11. ΔABC is a right-angled triangle in which $\angle A = 90°$, AC = 12 cm and BC = 13 cm. A circle with centre O has been inscribed inside the triangle.

Calculate the value of x, the radius of the inscribed circle.

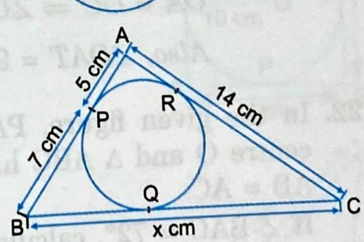


12. PQR is a right-angled triangle with PQ = 3 cm and QR = 4 cm. A circle which touches all the sides of the triangle is inscribed in the triangle. Calculate the radius of the circle. **(2005)**

13. In the given figure, O is the centre of each one of two concentric circles of radii 4 cm and 6 cm respectively. PA and PB are tangents to outer and inner circle respectively. If PA = 10 cm, find the length of PB, upto two places of decimal.

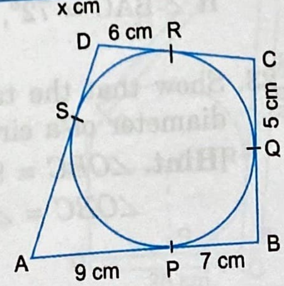


14. In the given figure, ΔABC is circumscribed. The circle touches the sides AB, BC and CA at P, Q, R respectively. If AP = 5 cm, BP = 7 cm, AC = 14 cm and BC = x cm, find the value of x.

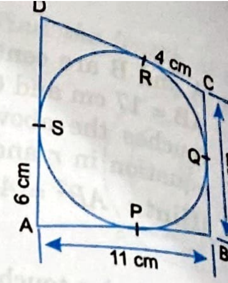


15. In the given figure, quadrilateral ABCD is circumscribed. The circle touches the sides AB, BC, CD and DA at P, Q, R, S respectively. If AP = 9 cm, BP = 7 cm, CQ = 5 cm and DR = 6 cm, find the perimeter of quadrilateral ABCD.

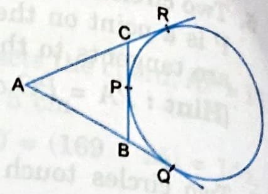
[**Hint.** *AP = AS, BP = BQ, CQ = CR and DR = DS.*]



16. In the given figure, the circle touches the sides AB, BC, CD and DA of a quadrilateral ABCD at the points P, Q, R and S respectively. If AB = 11 cm, BC = x cm, CR = 4 cm and AS = 6 cm, find the value of x.



17. In the given figure, a circle touches the side BC of ΔABC at P and AB and AC produced at Q and R respectively. If AQ = 15 cm, find the perimeter of ΔABC.



18. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 40°$, find $\angle AQB$ and $\angle AMB$.

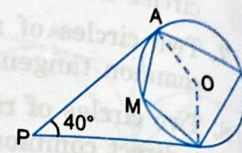
[**Hint.** *Join OA and OB.*

$\angle PAO = 90°$ *and* $\angle PBO = 90°$.

$\angle OAP + \angle APB + \angle PBO + \angle AOB = 360°$

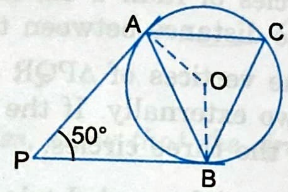
$\Rightarrow \angle AOB = 140°$.

$\therefore \angle AQB = 70°$ *and* $\angle AMB = (180° - 70°) = 110°$.]



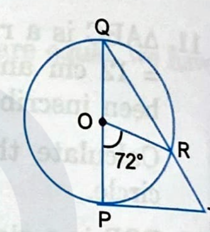
19. In the given figure, PA and PB are two tangents to the circle with centre O. If $\angle APB = 50°$, find :

(*i*) $\angle AOB$ (*ii*) $\angle OAB$ (*iii*) $\angle ACB$

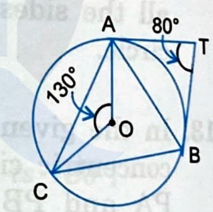


20. In the given figure, PQ is a diameter of a circle with centre O and PT is a tangent at P. QT meets the circle at R. If $\angle POR = 72°$, find $\angle PTR$.

[**Hint.** $\angle PQR = 36°$ *and* $\angle QPT = 90°$. *Sum of the* $\angle s$ *of* ΔQPT *is* 180°].



21. In the given figure, O is the centre of the circumcircle of ΔABC. Tangents at A and B intersect at T. If $\angle ATB = 80°$ and $\angle AOC = 130°$, calculate $\angle CAB$.



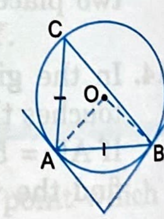
[**Hint.** $TA = TB \Rightarrow \angle TAB = \angle TBA = 50°$.

$OA = OC \Rightarrow \angle OAC = \angle OCA = 25°$

Also, $\angle OAT = 90°$. So $\angle OAB = (90° - 50°) = 40°$.]

22. In the given figure, PA and PB are tangents to a circle with centre O and Δ ABC has been inscribed in the circle such that AB = AC.

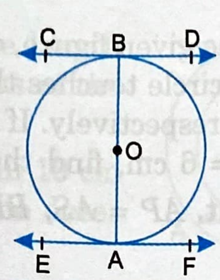
If $\angle$ BAC = 72°, calculate (*i*) $\angle$ AOB (*ii*) $\angle$ APB.



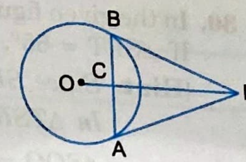
23. Show that the tangent lines at the end points of a diameter of a circle are parallel.

[**Hint.** $\angle OBC = 90°$ *and* $\angle OAF = 90°$.

$\angle OBC = \angle OAF$ *and these are alt.* $\angle s$.]



24. Prove that the tangents at the extremities of any chord make equal angles with the chord.

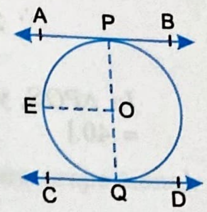


[*Hint.* Let the tangents at A and B meet at P. Join OP, meeting AB at C.

$\Delta PAC \cong \Delta PBC$ [$PA = PB$, $PC = PC$, $\angle APC = \angle BPC$

$\therefore \angle PAC = \angle PBC$.]

25. Show that the line segment joining the points of contact of two parallel tangents passes through the centre.



[*Hint.* Draw $EO \parallel AB \parallel CD$.

Now, $EO \parallel AB \Rightarrow \angle APO + \angle EOP = 180°$

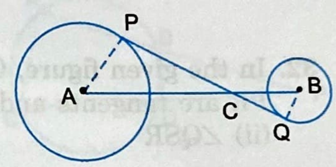
$\Rightarrow \angle EOP = 90°$ [$\because \angle APO = 90°$]

Similarly, $\angle EOQ = 90°$.

$\angle EOP + \angle EOQ = 180°$. So, POQ is a straight line.]

26. In the given figure, PQ is a transverse common tangent to two circles with centres A and B and of radii 5 cm and 3 cm respectively. If PQ intersects AB at C such that CP = 12 cm, calculate AB.

[*Hint.* Join AP and BQ. Then $AP \perp PQ$ and $BQ \perp PQ$.

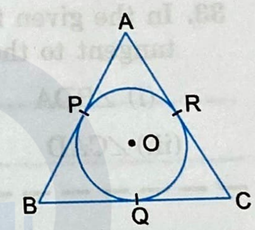


$\therefore CA = \sqrt{CP^2 + AP^2} = \sqrt{(12)^2 + (5)^2} = 13$ cm.

$\Delta PAC \sim \Delta QBC$ $\{\angle APC = \angle BQC = 90°, \angle PCA = \angle QCB\}$

$\therefore \dfrac{CA}{CB} = \dfrac{PA}{QB} \Leftrightarrow \dfrac{13}{CB} = \dfrac{5}{3} \Leftrightarrow CB = 7.8$ cm.]

27. ΔABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Prove that the base is bisected by the point of contact.



[*Hint.* AP = AR, BP = BQ and CQ = CR.

Now, AB = AC and AP = AR

$\Rightarrow AB - AP = AC - AR$

$\Rightarrow BP = CR \Rightarrow BQ = CQ$.]

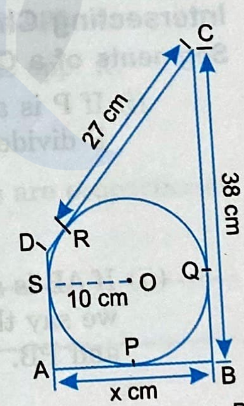
Lecture-V

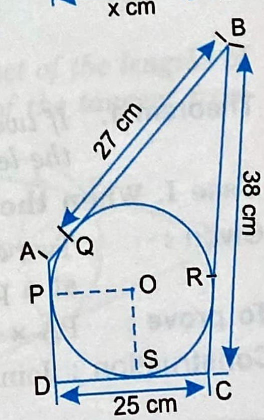
28. In the given figure, quadrilateral ABCD is circumscribed and $AD \perp AB$. If the radius of incircle is 10 cm, find the value of x.

[*Hint.* Let DR = DS = y cm. Also, AS = 10 cm.

Now, $AB + DC = BC + AD \Rightarrow x + 27 + y = 10 + y + 38$

$\Rightarrow x = 21$ cm.]





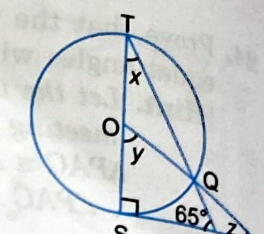
29. In the given figure, a circle is inscribed in quad. ABCD. If BC = 38 cm, BQ = 27 cm, DC = 25 cm and $AD \perp DC$, find the radius of the circle.

[*Hint.* BR = BQ = 27 cm. So, CR = (38 − 27) cm = 11 cm.

$\therefore$ CS = CR = 11 cm.

$\therefore$ DS = (25 − 11) cm = 14 cm = (Radius of the circle).]

30. In the given figure, O is the centre of the circle and SP is a tangent. If $\angle SRT = 65°$, find the values of x, y and z.



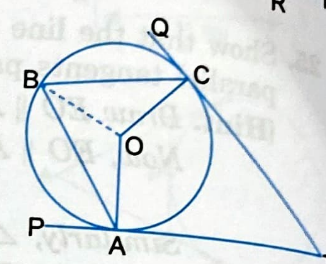
[Hint. *Since SP is a tangent at S, so* $\angle OSP = \angle TSP = 90°$.

In ΔTSR, $x° + 90° + 65° = 180° \Rightarrow x = 25°$.

$\angle SOQ = 2\angle STQ$ *[Angle at the centre is double the angle at a point on the remaining part of the circle]*

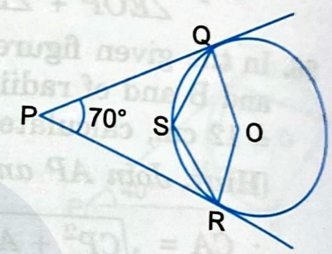
$\therefore\ y = 2x = 50°$

In ΔPOS, $y° + 90° + z° = 180° \Rightarrow 50° + 90° + z° = 180° \Rightarrow z = 40.]$



31. In the given figure, TP and TQ are two tangents to the circle with centre O, touching at A and C respectively. If $\angle BCQ = 55°$ and $\angle BAP = 60°$, find :

(i) $\angle OBA$ and $\angle OBC$ (ii) $\angle AOC$ (iii) $\angle ATC$ **(2020)**



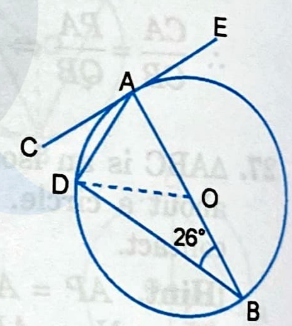
32. In the given figure, O is the centre of the circle. PQ and PR are tangents and $\angle QPR = 70°$. Calculate : (i) $\angle QOR$ (ii) $\angle QSR$

(2022, Semester 2)

33. In the given figure, O is the centre of the circle. CE is a tangent to the circle at A. If $\angle ABD = 26°$, then find :

(i) $\angle BDA$ (ii) $\angle BAD$

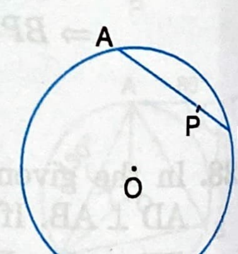
(iii) $\angle CAD$ (iv) $\angle ODB$ **(2023)**



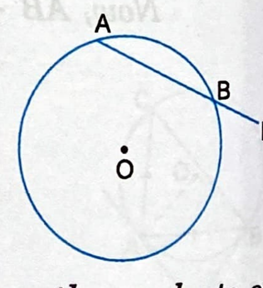
Intersecting Chords & Tangents

Segments of a Chord

(*i*) If P is a point on a chord AB of a circle, then we say that P divides AB internally into two segments PA and PB.



(*ii*) If AB is a chord of a circle and P is a point on AB produced, we say that P divides AB externally into two segments PA and PB.



Theorem 1. *If two chords of a circle intersect internally or externally, then the products of the lengths of their segments are equal.*

Case I. When the two chords intersect internally :

Given : Two chords AB and CD ...

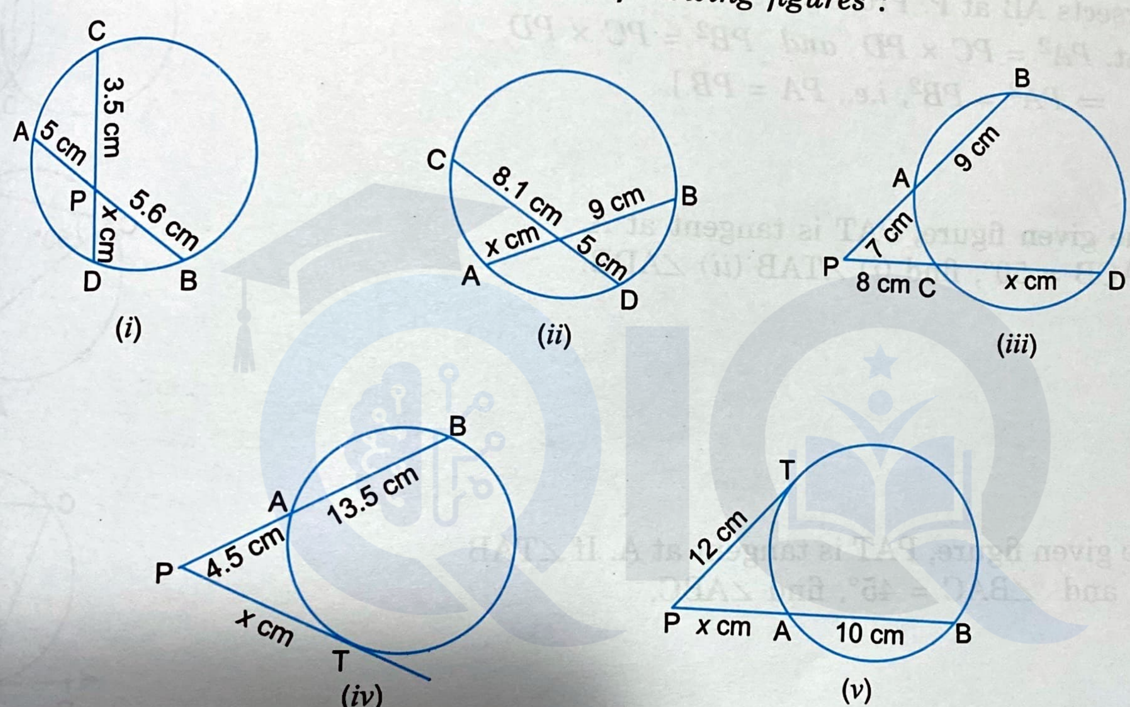
Exercise - 19B.

* Class-work → "✓" Marked questions.
* Home-work → "●" Bullet point questions.

$PA = PD.$

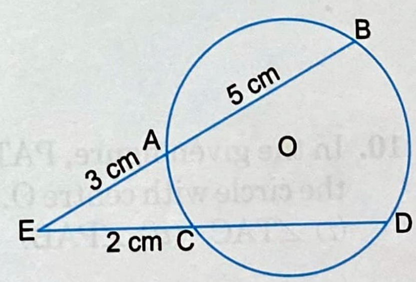
EXERCISE 19B

1. *Find the unknown length x in each of the following figures :*



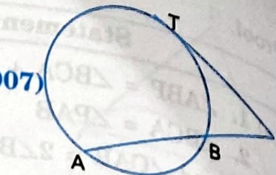
2. Two chords AB and CD of a circle intersect externally at E.

If EC = 2 cm, EA = 3 cm and AB = 5 cm, find the length of CD.



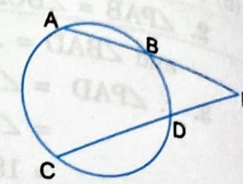
(2022, Semester 2)

3. In the adjoining figure, PT is a tangent to the circle. Find PT, if AP = 16 cm and AB = 12 cm. **(2007)**

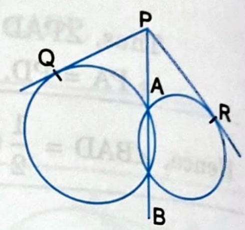


4. Two chords AB and CD of a circle intersect at a point P inside the circle such that AB = 12 cm, AP = 2.4 cm and PD = 7.2 cm. Find CD.
[Hint : *PA × PB = PC × PD.*]

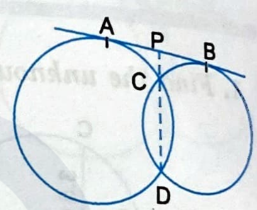
5. If AB and CD are two chords of a circle which when produced meet at a point P outside the circle such that PA = 12 cm, AB = 4 cm and CD = 10 cm, find PD.
[Hint. *PA = 12 cm, PB = (PA – AB) = 8 cm, PD = x cm,*
PC = (PD + CD) = (x + 10) cm.]



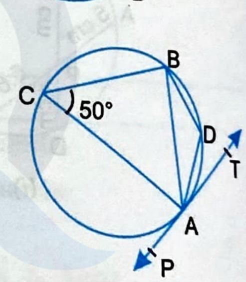
6. In the given figure, two circles intersect each other at the points A and B. If PQ and PR are tangents to these circles from a point P on BA produced, show that PQ = PR.
[Hint. $PQ^2 = PA \times PB$ *and* $PR^2 = PA \times PB$.]



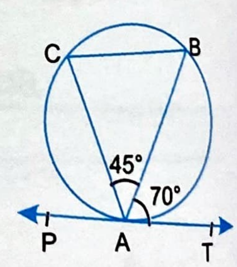
7. In the given figure, AB is a direct common tangent to two intersecting circles. Their common chord when produced, intersects AB at P. Prove that P is the mid-point of AB.
[Hint. $PA^2 = PC \times PD$ *and* $PB^2 = PC \times PD$
$\Rightarrow PA^2 = PB^2$, *i.e., PA = PB.*]



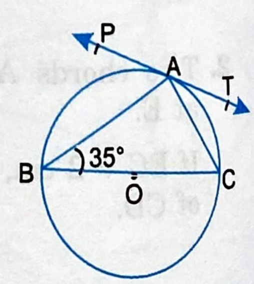
8. In the given figure, PAT is tangent at A. If ∠ACB = 50°, find (*i*) ∠TAB (*ii*) ∠ADB.



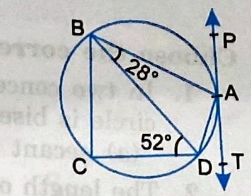
9. In the given figure, PAT is tangent at A. If ∠TAB = 70° and ∠BAC = 45°, find ∠ABC.



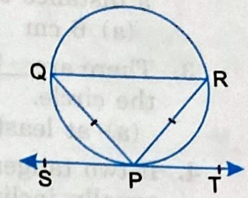
10. In the given figure, PAT is tangent at A, to the circle with centre O. If ∠ABC = 35°, find (*i*) ∠TAC (*ii*) ∠PAB.



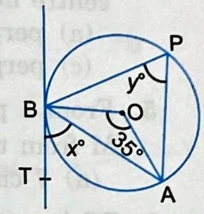
11. In the given figure, PAT is tangent at A and BD is a diameter of the circle. If $\angle ABD = 28°$ and $\angle BDC = 52°$, find:
(i) $\angle TAD$ (ii) $\angle BAD$ (iii) $\angle PAB$ (iv) $\angle CBD$.



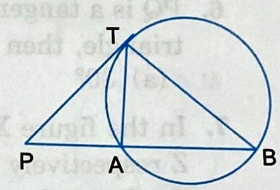
12. In the given figure, PQ and PR are two equal chords of a circle. Show that the tangent at P is parallel to QR.
[**Hint.** *$PQ = PR \Rightarrow \angle PRQ = \angle PQR$*
Also, $\angle TPR = \angle PQR$ [$\angle$s in alternate segments]
$\therefore \angle PRQ = \angle TPR$.]



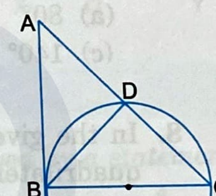
13. In the given figure, AB is a chord of the circle with centre O and BT is a tangent to the circle. If $\angle OAB = 35°$, find the values of x and y.



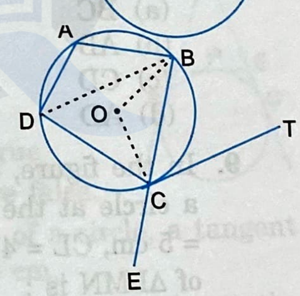
14. In the given figure, PAB is a secant to the circle and PT is a tangent at T.
Prove that :
(i) $\Delta PAT \sim \Delta PTB$
(ii) $PA \times PB = PT^2$.



15. In a right-angled ΔABC, the perpendicular BD on hypotenuse AC is drawn.
Prove that : (i) $AC \times AD = AB^2$ (ii) $AC \times CD = BC^2$.
[**Hint.** *Draw a circumcircle of ΔBCD.*
AB is a tangent & ADC is a secant $\Rightarrow AC \times AD = AB^2$. ...(i)
Now, $AC \times CD = AC \times (AC - AD)$
$= AC^2 - AC \times AD = AC^2 - AB^2 = BC^2$ [Using (i)]]



16. In the given figure, ABCD is a cyclic quadrilateral in which CB = CD and TC is a tangent to the circle at C. If O is the centre of the circle, BC is produced to E such that $\angle DCE = 110°$, find (i) $\angle DCT$ (ii) $\angle BOC$.
[**Hint.** *Join BD, OB and OC.*
$\angle DCE = 110° \Rightarrow \angle BCD = 70°$.
$CB = CD \Rightarrow \angle BDC = \angle DBC = 55°$.
$\therefore \angle BOC = (2 \times \angle BDC) = (2 \times 55°) = 110°$
Also, $\angle BCT = \angle DBC = 55°$ ($\angle$s in alt. segments)]



17. In the given figure, AC is a tangent to the circle with centre O. If $\angle ADB = 55°$, find x and y. Give reasons for your answers. **(2019)**