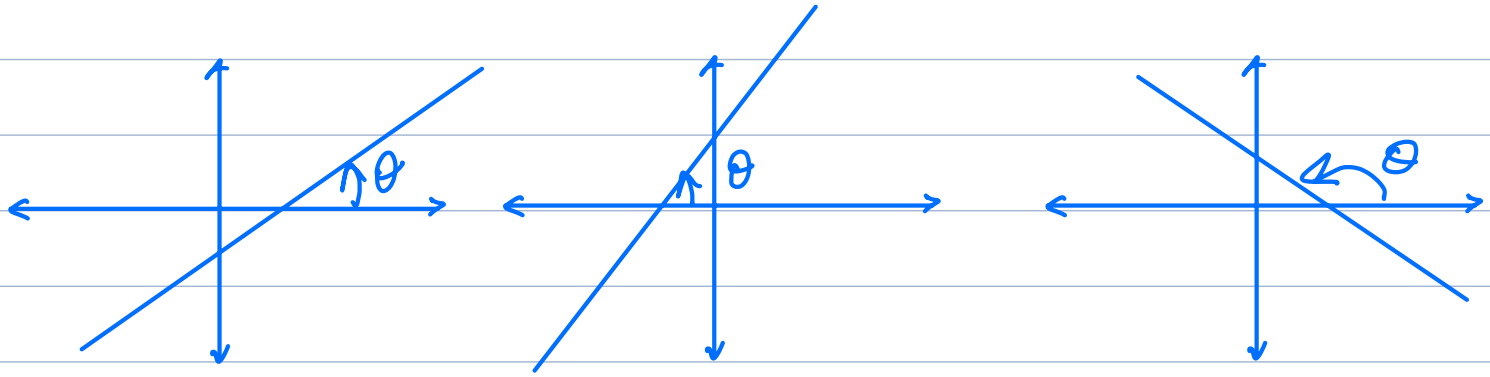
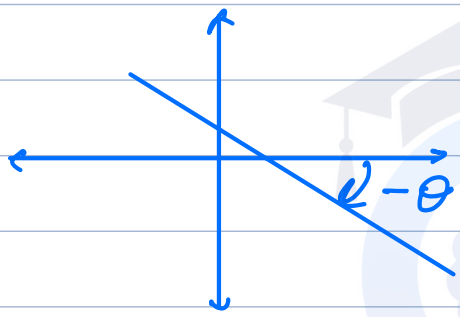
Straight lines. Total lecture = 3

Lecture-I

Angle of Inclination (θ) → Angle made by the line with x-axis in anticlockwise direction.





Note: i) Angle of inclination of any line parallel to x-axis or perpendicular to y-axis is $0°$.

ii) Angle of inclination of any line parallel to y-axis or perpendicular to x-axis is $90°$.

Slope or Gradient of a line. (m)

↳ If angle of inclination is θ then the slope m is $\tan\theta$.

e.g.: $\theta = 0^\circ \Rightarrow m = \tan 0^\circ = 0$

$\theta = 30^\circ \Rightarrow m = \tan 30^\circ = 1/\sqrt{3}$

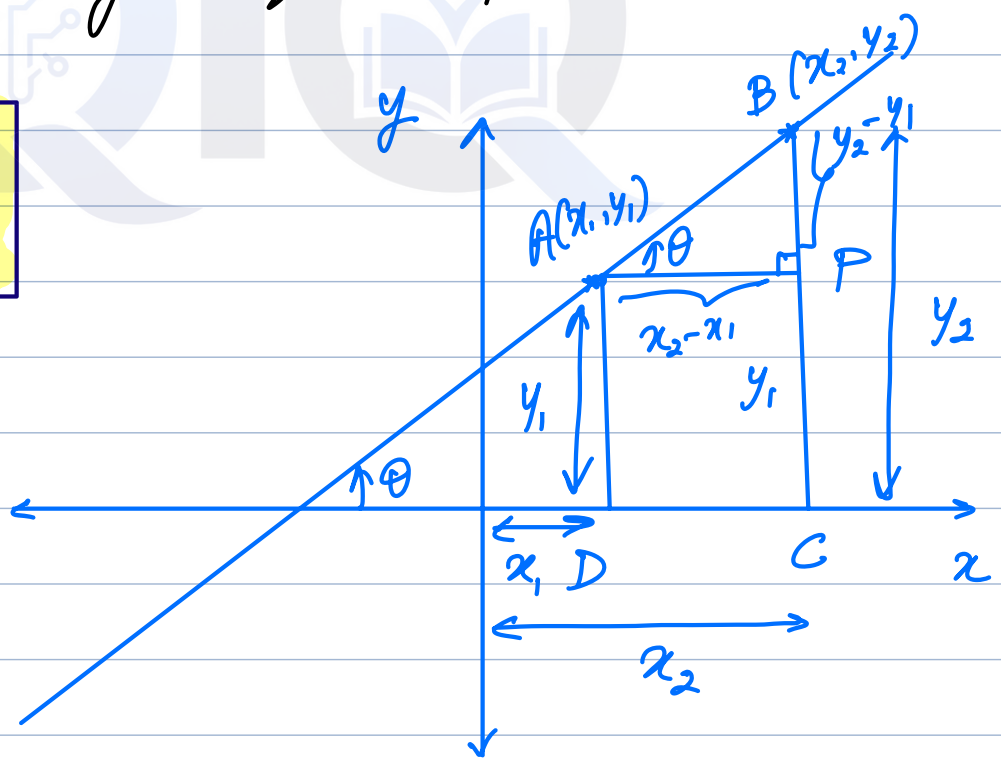
$\theta = 45^\circ \Rightarrow m = \tan 45^\circ = 1$

$\theta = 60^\circ \Rightarrow m = \tan 60^\circ = \sqrt{3}$

$\theta = 90^\circ \Rightarrow m = \tan 90^\circ =$ undefined (∞).

Slope of a line passing through two points $A(x_1, y_1)$ & $B(x_2, y_2)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Here:- $m = \tan\theta$

In $\triangle APB$

$$\tan\theta = \frac{BP}{AP} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1}.$$

e.g.: Find the slope of the line passing through the given below points.

i) $(0,0)$ & $(3,4)$ $\rightarrow$ $m = \frac{4-0}{3-0} = \frac{4}{3}$

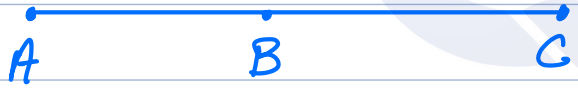
(x_1, y_1) (x_2, y_2)

ii) $(-1,1)$ & $(5,4)$ $\rightarrow$ $m = \frac{4-1}{5-(-1)} = \frac{3}{6} = \frac{1}{2}$

iii) $(2,-5)$ & $(-3,-4)$ $\rightarrow$ $m = \frac{-4+5}{-3-2} = \frac{1}{-5} = -\frac{1}{5}$.

Collinear points.

↳ Three or more than three points are said to Collinear if & only if they lie on the same line.



$\rightarrow$ A, B & C are Collinear.

$\Rightarrow$ **slope of AB = slope of BC**

e.g.: Find the value of K, when it is given that the points A(-1,1), B(5,7) & C(8,K) are Collinear.

Sol^{n}. Slope of AB = Slope of BC

$\Rightarrow \frac{7-1}{5-(-1)} = \frac{K-7}{8-5} \Rightarrow 1 = \frac{K-7}{3} \Rightarrow 3 = K-7$

$\Rightarrow$ **K = 10** Ans

Lecture-II

Various forms of a line.

1) General form of a line. → $ax+by+c=0$

a, b & c are constants.

e.g.: $x+y-1=0$, $2x-3y+5=0$ etc.

2) Slope-Intercept form → $y=mx+c$

↳ Equation of line having slope m & y-intercept be c.

3) Point-Slope form → $y-y_1 = m(x-x_1)$

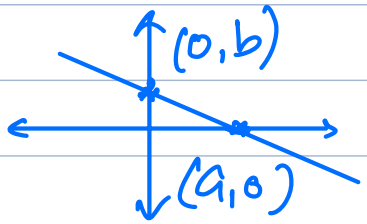
↳ Equation of line passing through a point (x_1,y_1) & having slope m.

4) Two point form → $y-y_1 = \left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)$

↳ Equation of line passing through two points (x_1,y_1) & (x_2,y_2)

5) Double Intercept form → $\frac{x}{a}+\frac{y}{b}=1$

↳ Equation of line having x-int & y-int to be a & b respectively.



How to find x-intercept & y-intercept of a line?

i) To find x-int. Substitute $y=0$ & solve for x.

ii) To find y-int. Substitute $x=0$ & solve for y.

e.g.: Find x-int & y-int.

(a) $3x-4y+12=0$ → for x-int, put $y=0$

$3x-4(0)+12=0 \Rightarrow 3x+12=0$

$\Rightarrow \boxed{x=-4}$ → x-int

for y-int, put $x=0$

$3(0)-4y+12=0 \rightarrow 4y=12 \Rightarrow \boxed{y=3}$ → y-int.

(b) $-x+y-9=0$ → x-int $=-9$ → $(-9,0)$

y-int $=9$ → $(0,9)$

How to find slope of the line $ax+by+c=0$?

$ax+by+c=0 \rightarrow by=-ax-c$

$y=\frac{-a}{b}x-\frac{c}{b}$ {$y=mx+c$}

$$\boxed{m=-\frac{a}{b}=-\frac{\text{Coefficient of } x}{\text{Coefficient of } y}}$$

e.g.: i) $3x+4y-1=0 \rightarrow m = -\frac{3}{4}$

ii) $x+y-5=0 \rightarrow m = -\frac{1}{1} = -1$

iii) $5x-3y+15=0 \rightarrow m = \frac{-5}{-3} = \frac{5}{3}$.

iv) $-9x+y-100=0 \rightarrow m = \frac{-(-9)}{1} = 9$

v) $2x-9=0 \rightarrow m = -\frac{(2)}{0} =$ undefined

vi) $3y+15=0 \rightarrow m = -\frac{(0)}{3} = 0$

Line is parallel to y-axis

Line is parallel to x-axis.

Condition of parallelism & perpendicularity.

* Let the line L_1 has slope m_1 & L_2 has slope m_2. then

(a) L_1 is parallel to L_2 if & only if $m_1 = m_2$

(b) L_1 is perpendicular to L_2 if & only if $m_1 m_2 = -1$

e.g.: $L_1: 3x + 4y - 5 = 0 \rightarrow m_1 = \frac{-3}{4}$

$L_1 \parallel L_2 \rightarrow m_2 = -3/4$

$L_1 \perp L_3 \rightarrow m_3 = \frac{4}{3}$

Illustration.

1) Find the value of p for which the lines $2x+3y-7=0$ & $4y-px-12=0$ are perpendicular to each other.

Soln: $L_1: 2x+3y-7=0 \longrightarrow m_1 = -\frac{2}{3}$

$L_2: 4y-px-12=0 \longrightarrow m_2 = \frac{-(-p)}{4} = \frac{p}{4}$

$\because L_1 \perp L_2 \therefore m_1 m_2 = -1 \Rightarrow -\frac{2}{3} \times \frac{p}{4} = -1 \Rightarrow \boxed{p=6}$ Ans

2) The line through the points $A(-2,3)$ & $B(4,b)$ is perpendicular to the line $2x-4y=5$. Find the value of b.

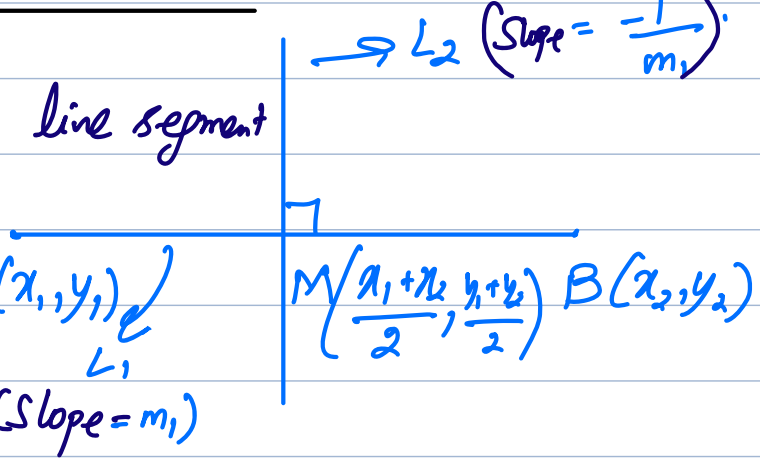
Soln: $L_1: \rightarrow m_1 = \frac{b-3}{4-(-2)} = \frac{b-3}{6}$.

$L_2: 2x-4y=5 \longrightarrow m_2 = \frac{-2}{-4} = \frac{1}{2}$

$\because L_1 \perp L_2 \therefore \frac{b-3}{6} \times \frac{1}{2} = -1 \Rightarrow b-3 = -12 \Rightarrow \boxed{b=-9}$

Right bisector or perpendicular bisector.

↳ A line which perpendicular to a line segment & passing through its mid-point.



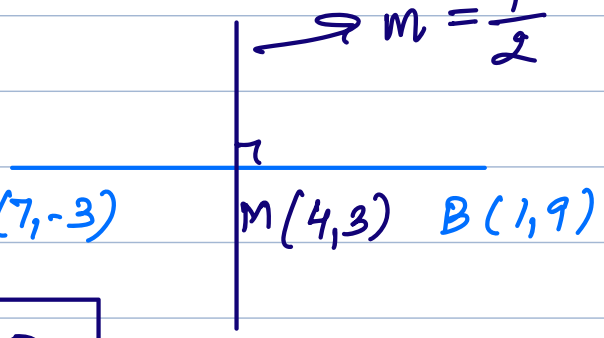
3) Point $A(7,-3)$ & $B(1,9)$ have been given. Find.

i) Slope of AB

ii) the equation of the perpendicular bisector of the line segment AB.

iii) the value of p if $(-2,p)$ lies on it.

i) Slope of $AB = \frac{9-(-3)}{1-7} = \frac{12}{-6} = -2$.



ii) $y-3 = \frac{1}{2}(x-4)$

$\Rightarrow 2y-6 = x-4 \Rightarrow \boxed{x-2y+2=0}$

iii) $-2-2p+2=0 \Rightarrow -2p=0 \Rightarrow \boxed{p=0}$ Ans

4) The equation of a line is $3x+4y-7=0$

Find the equation of a line perpendicular to the given line & passing through the intersection of the lines $x-y+2=0$ & $3x+y-10=0$.

Sol:

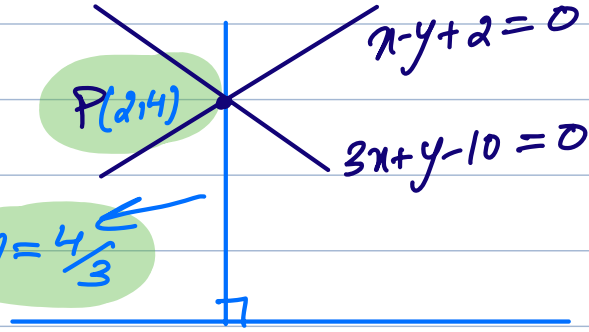
$$\begin{array}{r} x-y=-2 \\ +\ 3x+y=10 \\ \hline 4x=8 \end{array}$$

$\Rightarrow x=2$

$2-y=-2$

$2+2=y$

$\Rightarrow y=4$



$y-4 = \frac{4}{3}(x-2) \Rightarrow 3y-12 = 4x-8$

$\Rightarrow \boxed{4x-3y+4=0}$ Ans.

$3x+4y-7=0$

$m_1 = -3/4$

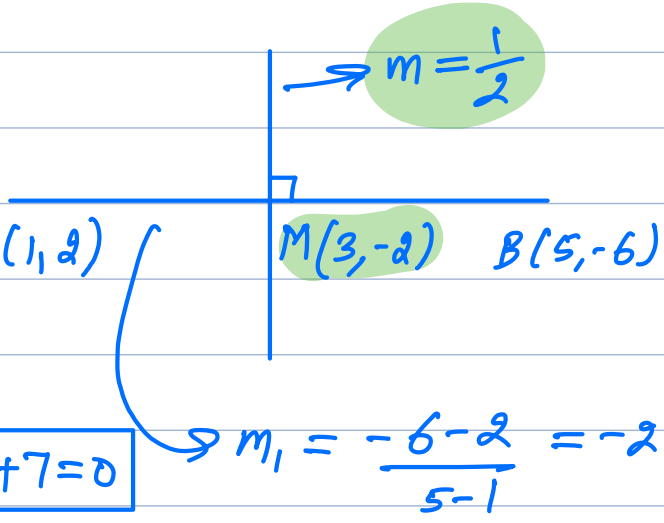
5) Find the equation of the right bisector of the line segment joining the points (1,2) & (5,-6).

Sol:- $\because\ y-y_1 = m(x-x_1)$

$\therefore\ y+2 = \frac{1}{2}(x-3)$

$\Rightarrow\ 2y+4 = x-3$

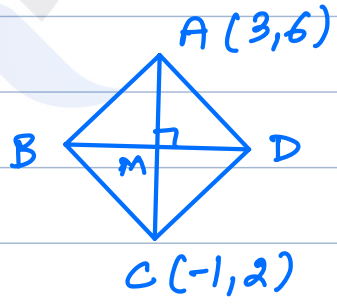
$\Rightarrow\ \boxed{x-2y-7=0}$ or $\boxed{2y-x+7=0}$



Lecture-III

Note:- Diagonals of a Rhombus bisects each other at 90°.

6) ABCD is a rhombus. The coordinates of A & C are (3,6) & (-1,2) respectively. Write down the equation of BD.



Sol:- M(1,4)

Slope of $AC = \frac{6-2}{3-(-1)} = 1.$

$\Rightarrow$ Slope of BD = -1

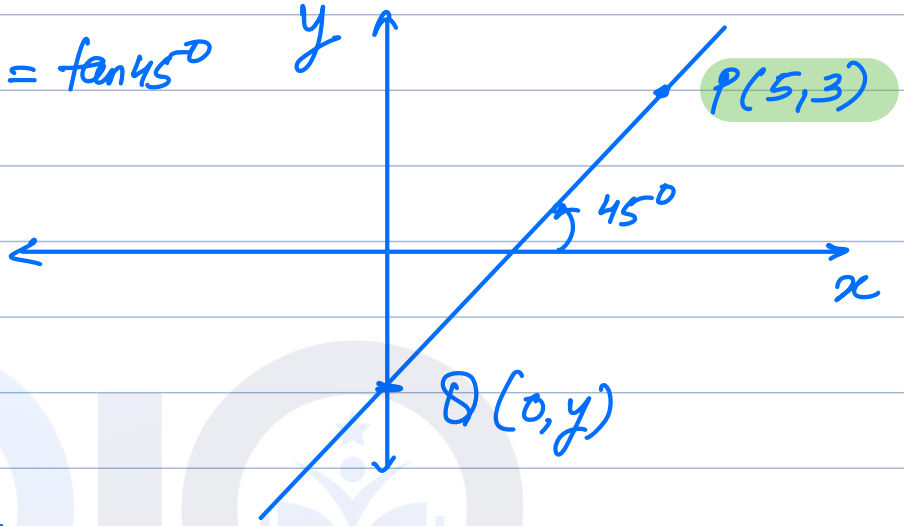
$y-4 = -1(x-1) \Rightarrow y-4 = -x+1 \Rightarrow \boxed{x+y-5=0}$ Ans

7) The line through a point $P(5,3)$ intersect y-axis at Q, making an angle of 45° with the x-axis.

i) write the slope of the line PQ.

ii) write the equation of the line PQ

iii) find the coordinate of Q.



Sol:- i) Slope of the line PQ = $\tan 45^\circ$

$\Rightarrow \boxed{m = 1}$

ii) $\because y - y_1 = m(x - x_1)$

$\therefore y - 3 = 1(x - 5)$

$\Rightarrow \boxed{x - y - 2 = 0}$ Ans.

iii) for y-int; put $x = 0 \Rightarrow 0 - y - 2 = 0 \Rightarrow \boxed{y = -2}$

$\Rightarrow Q(0, -2)$ Ans

8) A & B are two points on the x-axis & y-axis respectively & P(2, -3) is the mid-point of AB.

Find i) the coordinates of A & B.

ii) slope of the line AB.

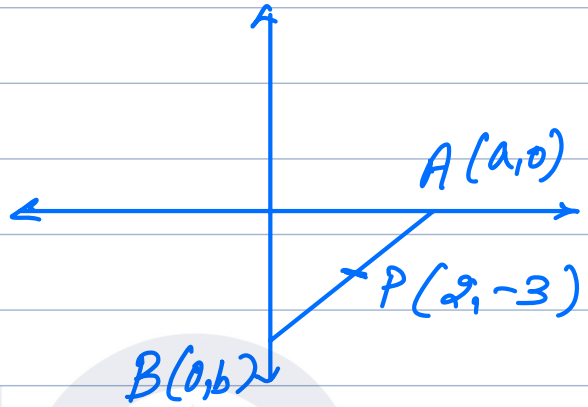
iii) equation of line AB.

Sol^n: i)

$$2 = \frac{a+0}{2} \Rightarrow a = 4$$

& $$-3 = \frac{b+0}{2} \Rightarrow b = -6$$

$\Rightarrow$ A(4, 0) & B(0, -6)



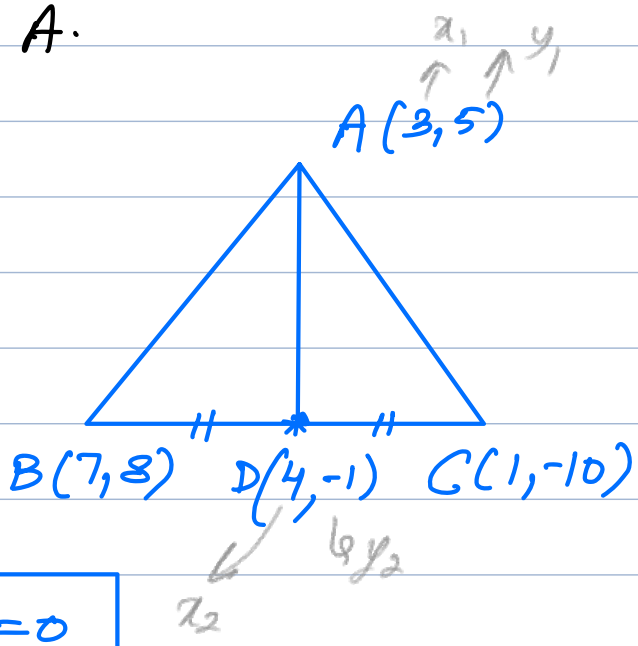
ii) Slope of line AB $= \frac{-6-0}{0-4} = \frac{3}{2} = m$

iii) eq^n of line line :- $y - 0 = \frac{3}{2}(x-4)$

$2y = 3x - 12 \Rightarrow$ $3x - 2y - 12 = 0$

Ans

9) In $\triangle ABC$, $A(3,5)$, $B(7,8)$ & $C(1,-10)$ are the vertices. Find the equation of Median through A.



Sol^n: $\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\therefore y - 5 = \frac{-1-5}{4-3}(x-3)$

$y - 5 = -6(x-3)$

$y - 5 = -6x + 18 \Rightarrow \boxed{6x + y - 23 = 0}$

Ans

10) Find the equation of a line passing through the point $(-2,3)$ & having x-intercept of 4 units.

$(-2,3)$: x_1, y_1

x-intercept of 4 units $\Rightarrow (4,0)$: x_2, y_2.

Sol^n:-

$\because \; y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1) \quad \therefore \; y-3 = \frac{0-3}{4+2}(x+2)$

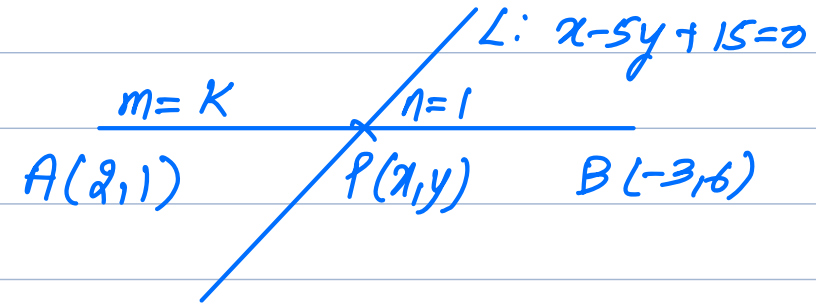
$\Rightarrow \; y-3 = \frac{-1}{2}(x+2) \Rightarrow 2y-6 = -x-2$

$\Rightarrow \boxed{x+2y-4=0}$ Ans

14(A) (25)

In what ratio the line $x-5y+15=0$ divide the join of $A(2,1)$ & $B(-3,6)$? Also find the co-ordinates of their point of intersection.

Sol:- Let the line $x-5y+15=0$ divides AB in the ratio $K:1$



$\because x = \frac{mx_2+nx_1}{m+n}$ & $y = \frac{my_2+ny_1}{m+n}$

$\therefore x = \frac{-3K+2}{K+1}$ & $y = \frac{6K+1}{K+1}$

$\because$ P lies on the line $x-5y+15=0$

$\therefore \frac{-3K+2}{K+1} - 5\left(\frac{6K+1}{K+1}\right) + 15 = 0$

$\Rightarrow -3K+2-30K-5+15K+15 = 0$

$\Rightarrow -18K+12 \Rightarrow 18K = 12 \Rightarrow K = 2/3$.

$\Rightarrow$ Required ratio is $\frac{2}{3}:1$ or $2:3$.

$x = \frac{-3\times\frac{2}{3}+2}{\frac{2}{3}+1} = \frac{-6+6}{5} = 0$

$y = \frac{6\times\frac{2}{3}+1}{\frac{2}{3}+1} = \frac{15}{5} = 3.$

$\therefore P(0,3)$

Ans

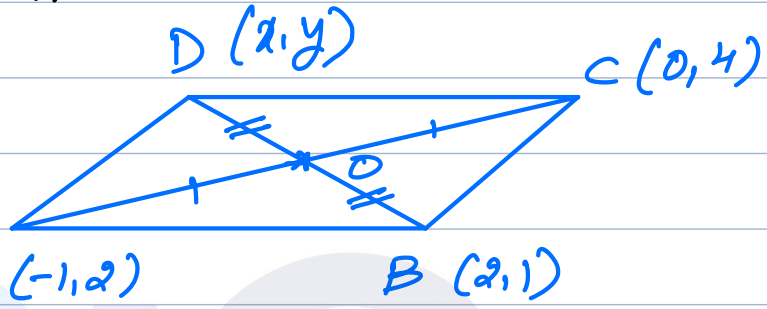
14(A) 27.

Three vertices of a parallelogram ABCD is taken in order are A(-1,2), B(2,1) & C(0,4). Hence, find

i) the co-ordinates of the fourth vertex D.

ii) length of Diagonal BD.

iii) equation of side AB.



Sol^n: ∵ Diagonal of ||gm bisects each other.

∴ O is the midpoint of AC as well as of BD

$$\Rightarrow O\left(\frac{-1}{2}, 3\right) = \left(\frac{x+2}{2}, \frac{y+1}{2}\right)$$

$$\Rightarrow \frac{x+2}{2} = \frac{-1}{2} \quad \& \quad \frac{y+1}{2} = 3$$

$$x = -3 \quad \& \quad y = 5 \Rightarrow D(-3, 5)$$ Ans