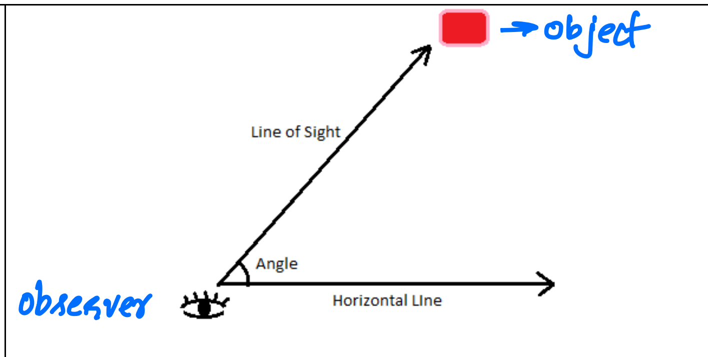
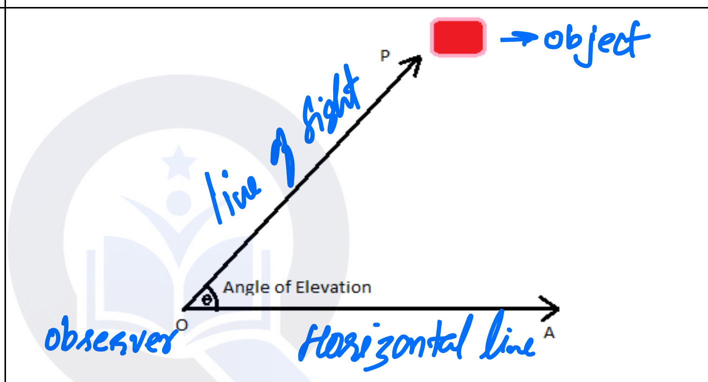
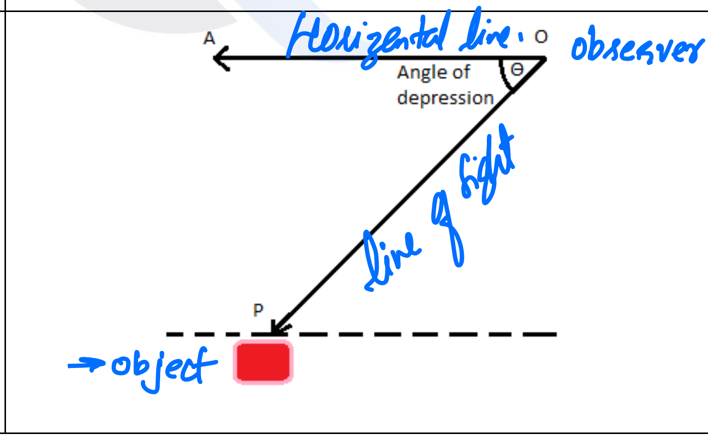
Total no. of lectures = 2.

SOME APPLICATIONS OF TRIGONOMETRY

HEIGHT AND DISTANCES

Lecture-I

KEY POINTS

Line of sight Line segment joining the object to the eye of the observer is called the line of sight.	
Angle of elevation When an observer sees an object situated in upward direction, the angle formed by line of sight with horizontal line is called angle of elevation.	
Angle of depression When an observer sees an object situated in downward direction the angle formed by line of sight with horizontal line is called angle of depression.	

Important Basic angles.

$\sin 30^\circ = \frac{1}{2}$, $\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 60^\circ = \frac{\sqrt{3}}{2}$.

$\cos 30^\circ = \frac{\sqrt{3}}{2}$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\cos 60^\circ = \frac{1}{2}$.

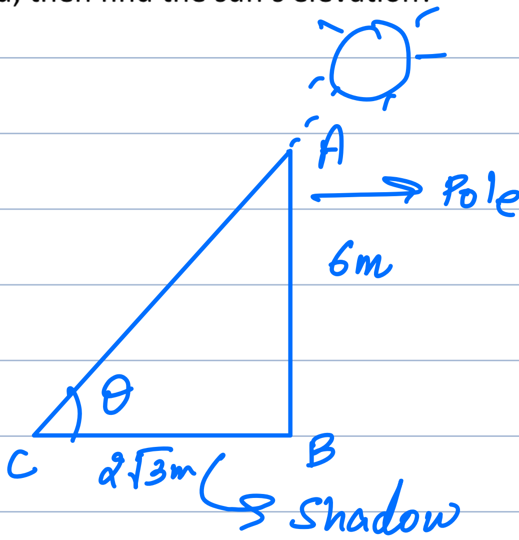
$\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\tan 45^\circ = 1$, $\tan 60^\circ = \sqrt{3}$.

$\frac{a}{\sqrt{a}} = \sqrt{a}$ $\left\{ a = \sqrt{a}\cdot\sqrt{a} \quad \frac{a}{\sqrt{a}} = \frac{\sqrt{a}\cdot\sqrt{a}}{\sqrt{a}} = \sqrt{a} \right\}$

1. A pole 6 m high casts a shadow $2\sqrt{3}$m long on the ground, then find the sun's elevation?

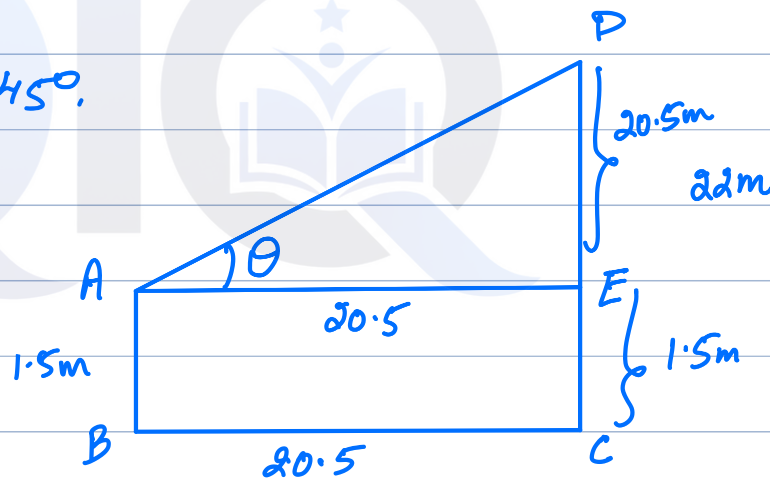
In $\triangle ABC \Rightarrow \tan\theta = \frac{\cancel{6}\,3}{\cancel{2}\sqrt{3}} = \sqrt{3}$

$\theta = 60^0$ Ans



2. An observer 1.5m tall is 20.5 metres away from a tower 22m high. Determine the angle of elevation of the top of the tower from the eye of the observer.

$\tan\theta = \frac{20.5}{20.5} = 1 \Rightarrow \theta = 45^0.$



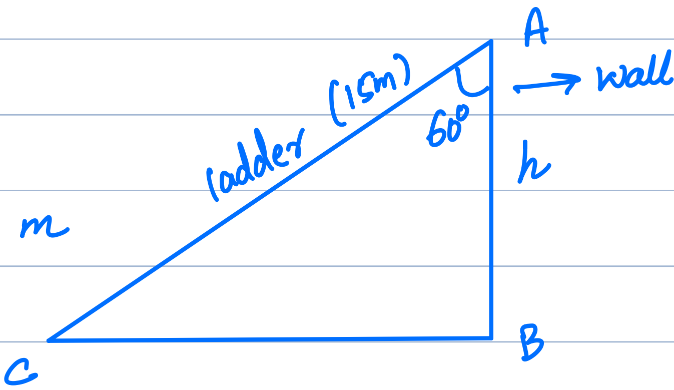
3. A ladder 15m long just reaches the top of vertical wall. If the ladder makes an angle 60^0 with the wall, find the height of the wall
(h)

$\cos 60^0 = \frac{h}{15}$

$\frac{1}{2} = \frac{h}{15} \Rightarrow h = \frac{15}{2}$ m

or

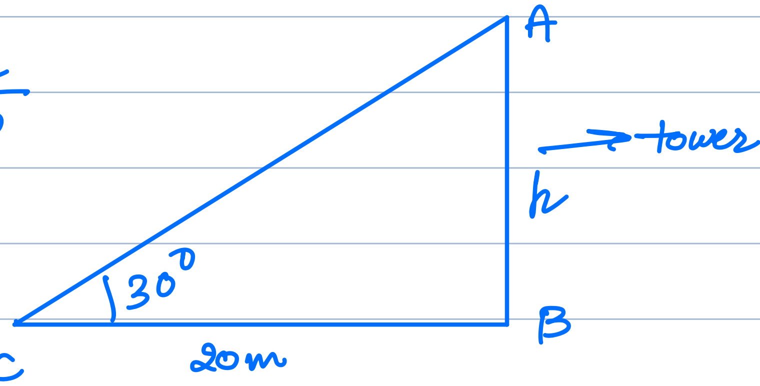
$h = 7.5m$ Ans.



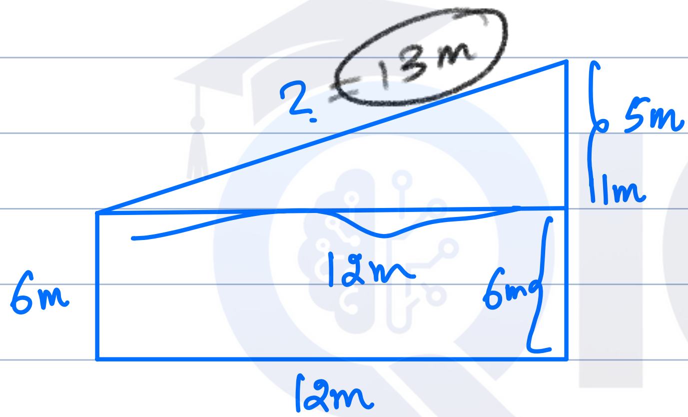
4. From a point 20m away from the foot of a tower, the angle of elevation of top of the tower is 30°, find the height of the tower.

$\tan 30^\circ = \frac{h}{20} \rightarrow \frac{1}{\sqrt{3}} = \frac{h}{20}$

$\Rightarrow h = \frac{20}{\sqrt{3}}\, m = \frac{20\sqrt{3}}{3}\, m$



5. Two poles of height 6m and 11m stands vertically on the ground. If the distance between their feet is 12m. Find the distance between their tops.



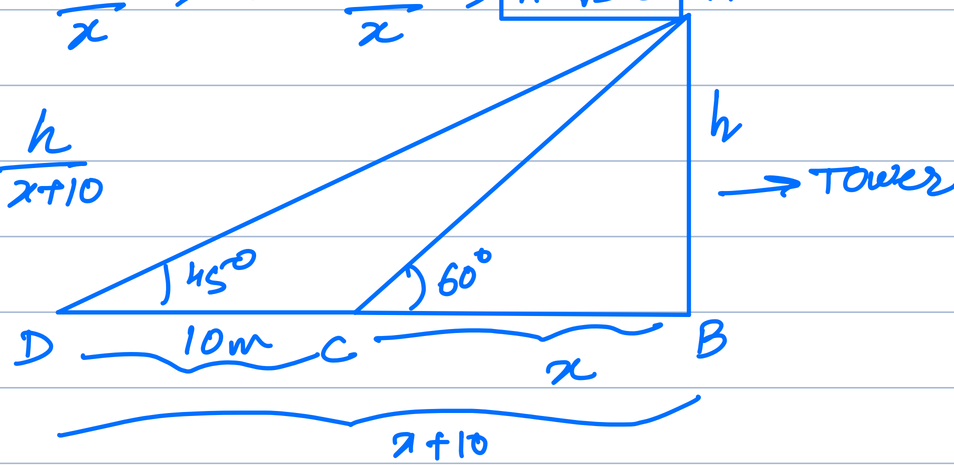
6. The shadow of tower, when the angle of elevation of the sun is 45° is found to be 10m longer than when it is 60°. Find the height of the tower.

Sol:

* In $\triangle ABC$, $\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow \boxed{h = \sqrt{3}x}$

* In $\triangle ABD$, $\tan 45^\circ = \frac{h}{x+10}$

$\Rightarrow 1 = \frac{h}{x+10}$



$\Rightarrow x + 10 = h$

$\Rightarrow x + 10 = \sqrt{3}x$

$\Rightarrow 10 = \sqrt{3}x - x \Rightarrow x(\sqrt{3}-1) = 10 \Rightarrow x = \frac{10}{\sqrt{3}-1} \Rightarrow x = \frac{10(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$$x = \frac{10(\sqrt{3}+1)}{2} \Rightarrow x = 5(\sqrt{3}+1)\,m$$

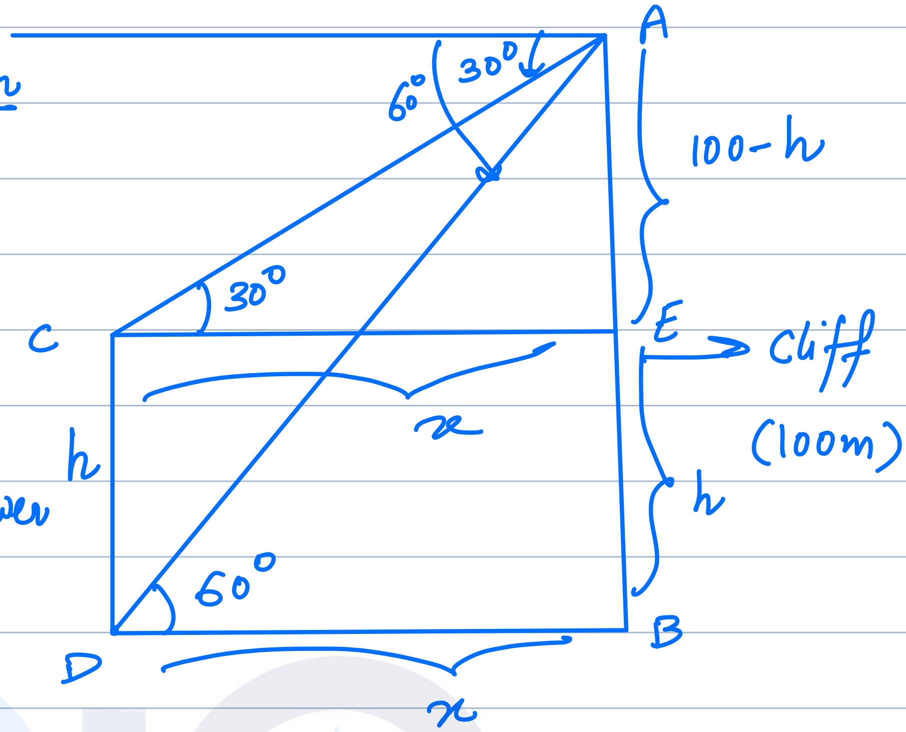
$$\Rightarrow h = \sqrt{3}x \Rightarrow h = 5\sqrt{3}(\sqrt{3}+1)\,m$$

$$\boxed{h = 5(3+\sqrt{3})\,m}$$ Ans

$$h = 5(3+1.732)\,m = 23.66\,m$$ Ans

$\sqrt{2} = 1.414$

1) . The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30^0 and 60^0 respectively. Find the height of the tower.



Sol:

In $\triangle ACE$, $\tan 30^0 = \dfrac{100-h}{x}$

$$\frac{1}{\sqrt{3}} = \frac{100-h}{x}$$

$$\boxed{x = \sqrt{3}(100-h)} \rightarrow (1)$$

In $\triangle ABD$

$\tan 60^0 = \dfrac{100}{x}$

$$\boxed{x = \frac{100}{\sqrt{3}}} \rightarrow (2)$$

from (1) & (2) $\dfrac{100}{\sqrt{3}} = \sqrt{3}(100-h) \rightarrow 100 = 3(100-h)$

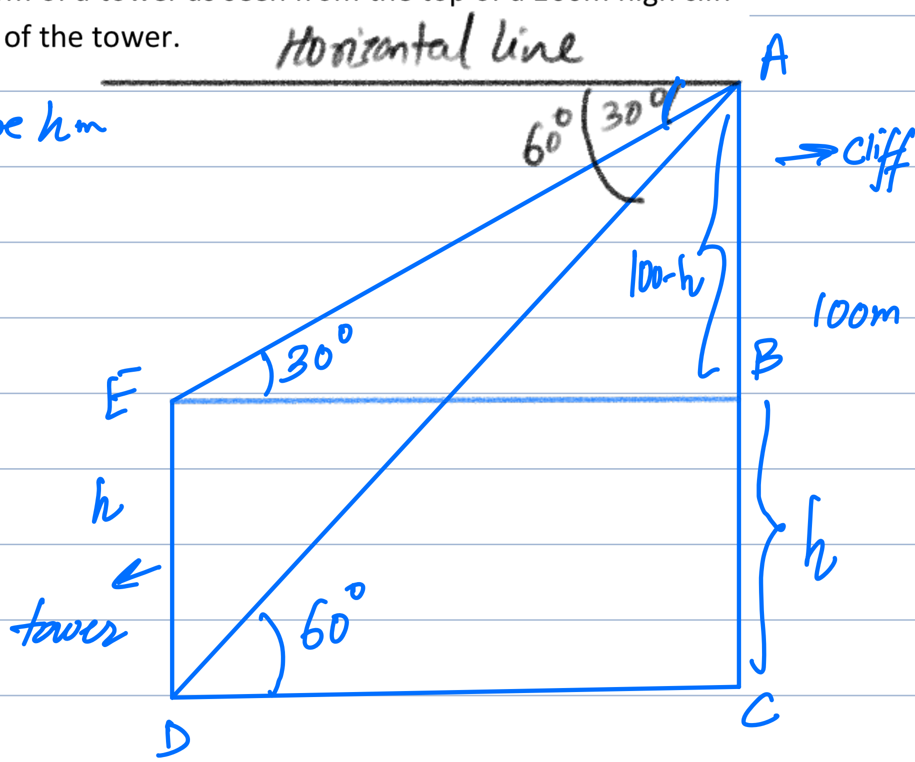
$$100 = 300 - 3h$$

$$3h = 200$$

$$h = \frac{200}{3} m = \underline{\underline{66.67 m}}$$

Lecture-II

14. The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30^0 and 60^0 respectively. Find the height of the tower.



Sol^n: Let the height of the tower be h m

In $\triangle ACD$, $\tan 60^0 = \frac{100}{CD}$

$$\boxed{CD = \frac{100}{\sqrt{3}} \text{ m} = BE}$$

In $\triangle ABE$, $\tan 30^0 = \frac{100-h}{BE}$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{100-h}{\frac{100}{\sqrt{3}}} \Rightarrow 100-h = \frac{1}{\sqrt{3}} \cdot \frac{100}{\sqrt{3}} = \frac{100}{3}$$

$$h = 100 - \frac{100}{3} = \frac{200}{3} \text{ m.}$$

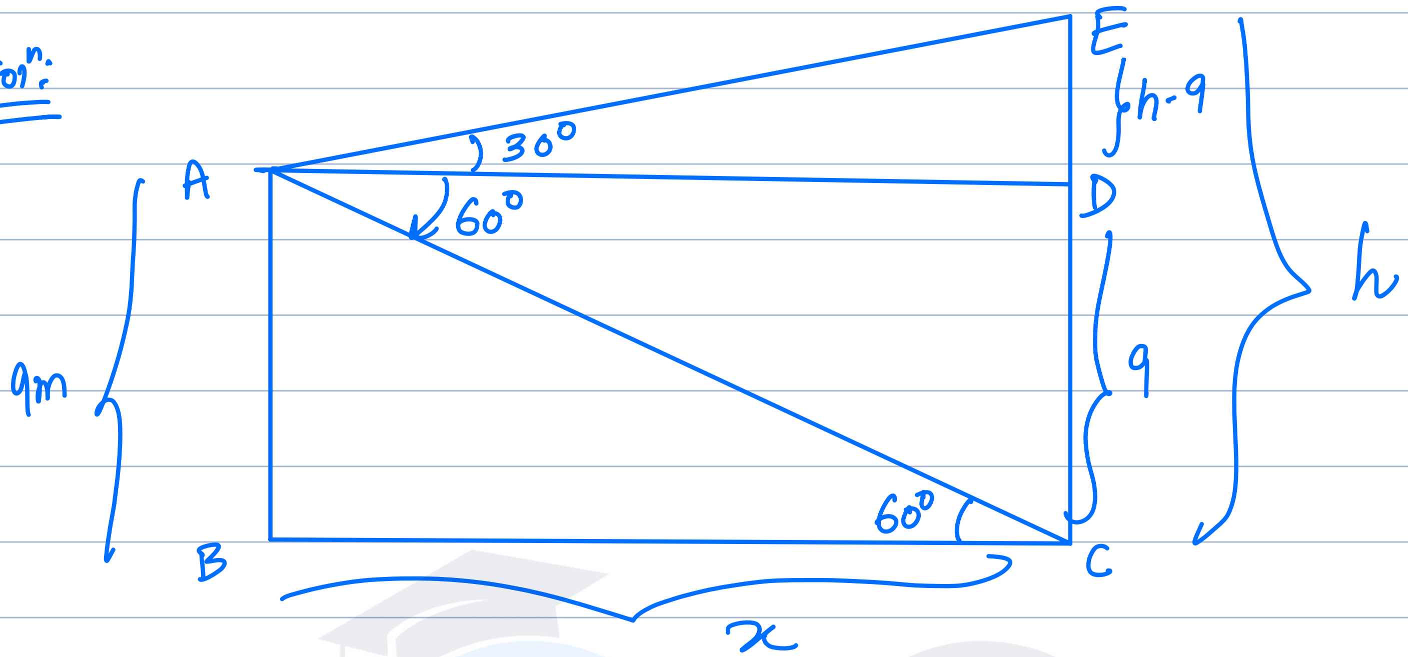
1) . The angle of depression of the top and bottom of a tower as seen from the top of a 100m high cliff are 30^0 and 60^0 respectively. Find the height of the tower.

2) From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

3) From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0. Find the height of the hill.

2) From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

Sol^n:

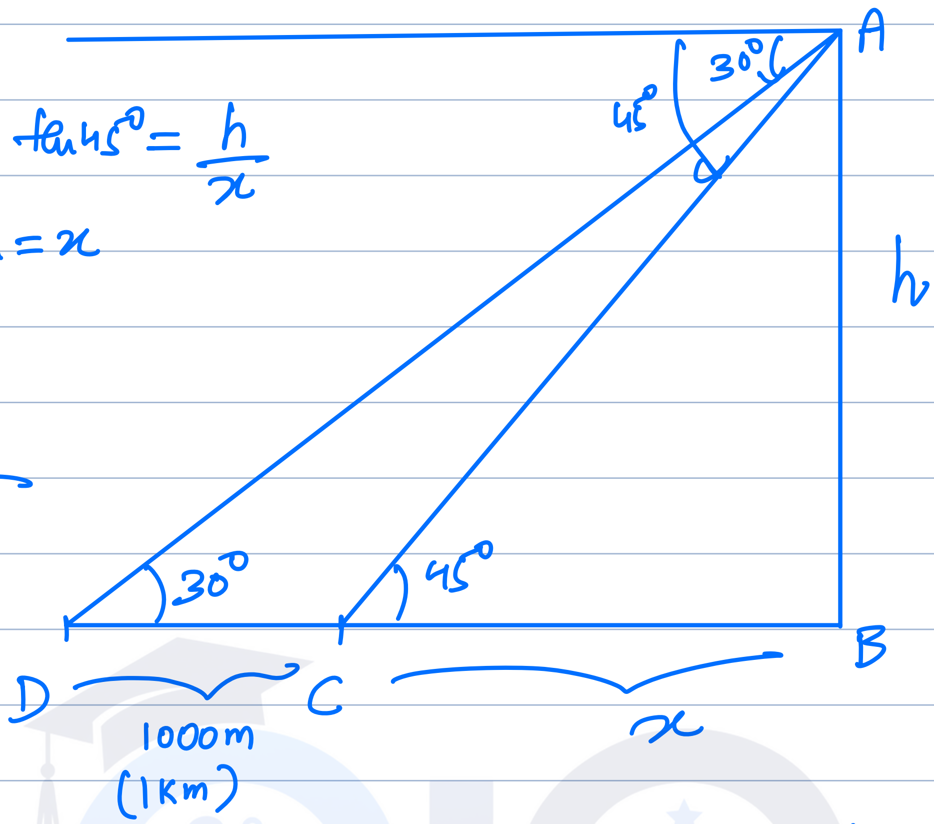


In $\triangle ABC$, $\tan 60^\circ = \dfrac{9}{x} \rightarrow x = \dfrac{9}{\sqrt{3}}\,m = \dfrac{9\sqrt{3}}{3}\,m = \underline{3\sqrt{3}\,m}$.

In $\triangle ADE$, $\tan 30^\circ = \dfrac{h-9}{x} \rightarrow \dfrac{1}{\cancel{\sqrt{3}}} = \dfrac{h-9}{3\cancel{\sqrt{3}}} \rightarrow h-9 = 3$

$h = \underline{12m}$ Ans

3) From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0. Find the height of the hill.



In $\triangle ABC$, $\tan 45^0 = \frac{h}{x}$

$h = x$

In $\triangle ABD$,

$\tan 30^0 = \frac{h}{x+1}$

$\frac{1}{\sqrt{3}} = \frac{h}{h+1}$

$h+1 = \sqrt{3}h$

$\sqrt{3}h - h = 1 \rightarrow h = \frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$h = \frac{\sqrt{3}+1}{2} = \frac{1.732+1}{2} = \frac{2.732}{2}$

$= \underline{1.366 \text{ Km}} = \underline{1366\text{m}}$

8. ~~15~~ From a window (9m above ground) of a house in a street, the angles of elevation and depression of the top and foot of another house on the opposite side of the street are 30^0 and 60^0 respectively. Find the height of the opposite house and width of the street.

Sol^n:

In $\triangle ABE$;

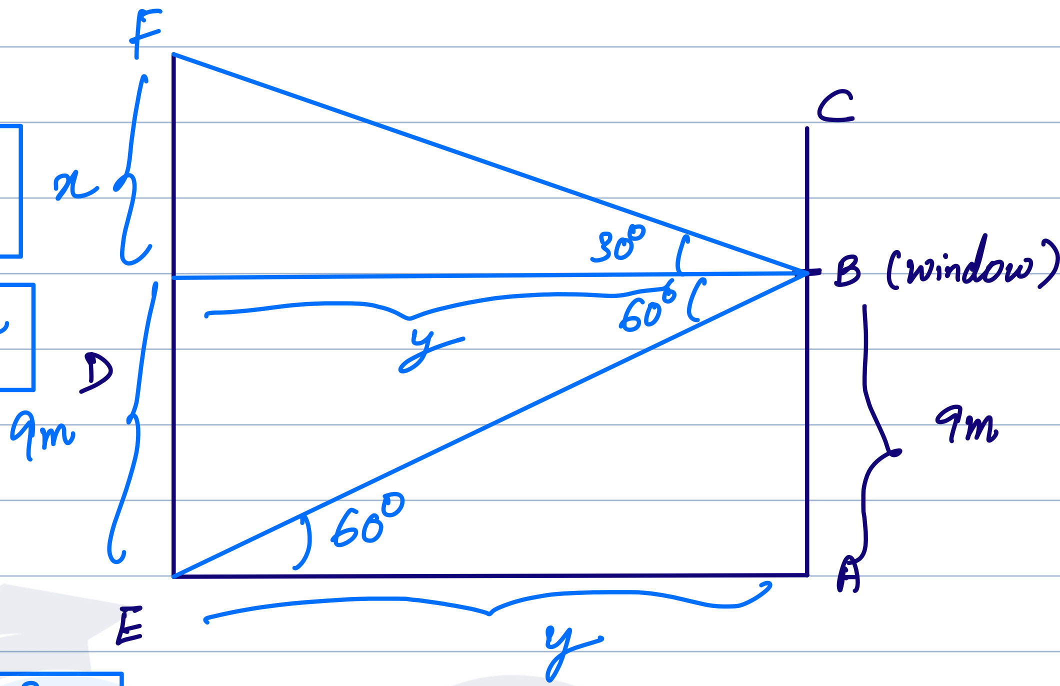
$\tan 60^0 = \frac{9}{y} \rightarrow \boxed{y = \frac{9}{\sqrt{3}}}$

$y = \frac{9\sqrt{3}}{3} \Rightarrow \boxed{y = 3\sqrt{3}\,m}$

In $\triangle BDF$

$\tan 30^0 = x/y$

$\frac{1}{\sqrt{3}} = \frac{x}{3\sqrt{3}} \Rightarrow \boxed{x = 3m}$



Height of the opposite house = 12m.
width of the street = $3\sqrt{3}$ m.

9. ~~16~~. From the top of a hill, the angle of depression of two consecutive kilometer stones due east are found to be 30^0 and 45^0. Find the height of the hill.

Sol^n:

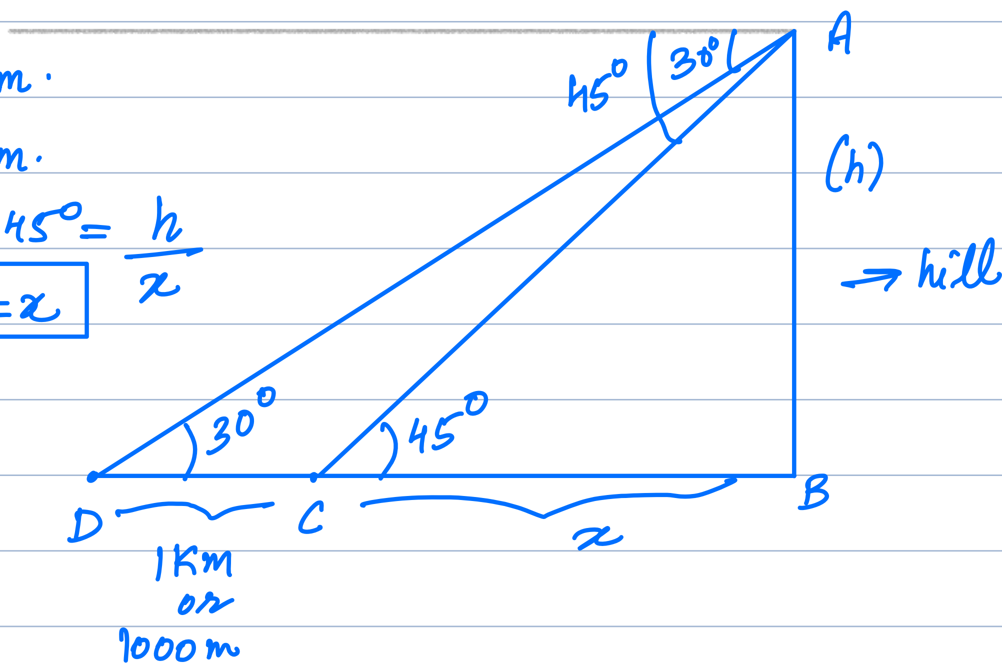
Let AB to be h m.
& BC to be x m.

In $\triangle ABC \rightarrow \tan 45^0 = \frac{h}{x}$

$\Rightarrow 1 = \frac{h}{x} \Rightarrow \boxed{h = x}$

In $\triangle ABD$

$\tan 30^0 = \frac{h}{x + 1000}$



$$\frac{1}{\sqrt{3}} = \frac{h}{x+1000} \Rightarrow x+1000 = \sqrt{3}h \Rightarrow h+1000 = \sqrt{3}h$$

$$\Rightarrow \sqrt{3}h - h = 1000 \Rightarrow h = \frac{1000}{\sqrt{3}-1}$$

$$\Rightarrow h = \frac{1000(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{1000(\sqrt{3}+1)}{2}$$

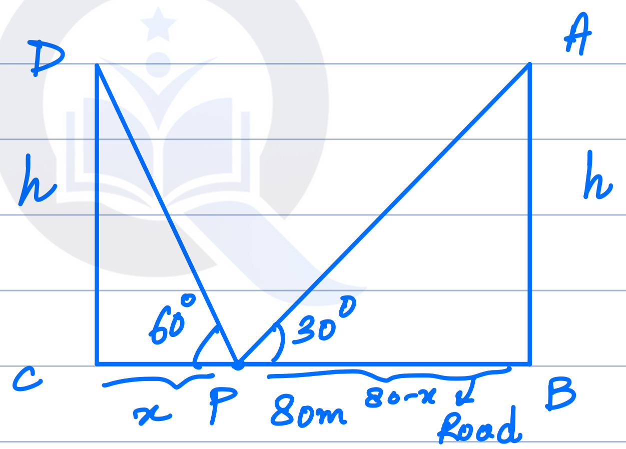
$$h = 500(\sqrt{3}+1)\,m \quad \text{Ans}$$

$$h = 500(1.732+1)m = 1366m.$$

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80m wide. From a point between them on the road the angles of elevation of the top of the poles are 60◦ and 30◦. Find the heights of pole and the distance of the point from the poles.

Let the heights of the pole be hm.

& CP be xm then BP = (80-x)m.



In $\triangle CDP$:- $\tan 60° = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

or $x = \frac{h}{\sqrt{3}}$

In $\triangle ABP$:- $\tan 30° = \frac{h}{80-x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{80-x}$.

$80 - x = \sqrt{3}h$

$\Rightarrow 80 - x = \sqrt{3}(\sqrt{3}x) \Rightarrow 80 - x = 3x \Rightarrow 80 = 4x \Rightarrow x = 20m$

$\Rightarrow h = 20\sqrt{3}\,m \Rightarrow h = 20(1.732)m = 34.64m.$

$\Rightarrow$ heights of the pole = 34.64m & distance of the point from the poles are 20m & 60m respectively

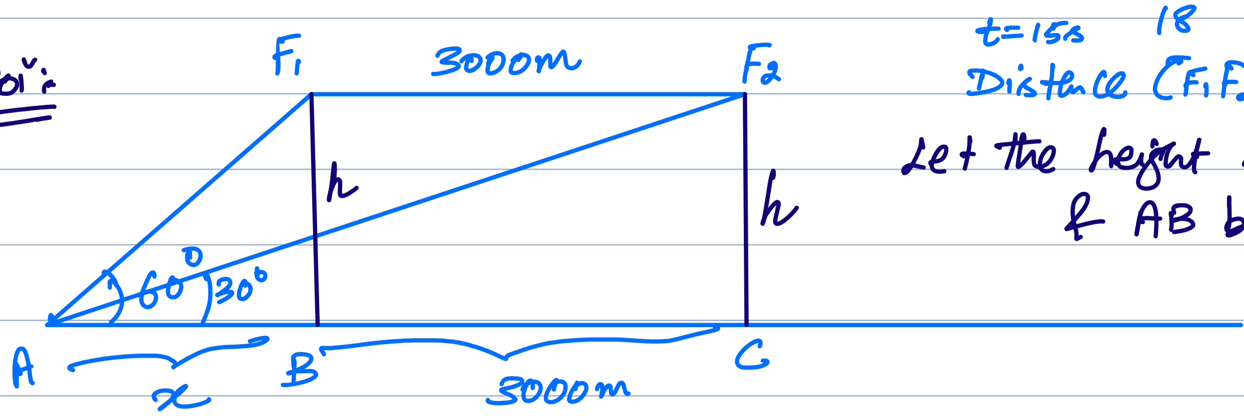
(18) The angle of elevation of a jet fighter from a point A on the ground is 60^0. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet is flying at a speed of 720km/ hr, find the constant height at which the jet is flying.

$S = 720 \times \frac{5}{18} = 200\,m/s$

$t = 15s$

Distance $(F_1F_2) = 3000\,m$

Sol^n:



Let the height be h m.
& AB be x m.

In $\triangle ABF_1$; $\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow \boxed{h = \sqrt{3}x}$

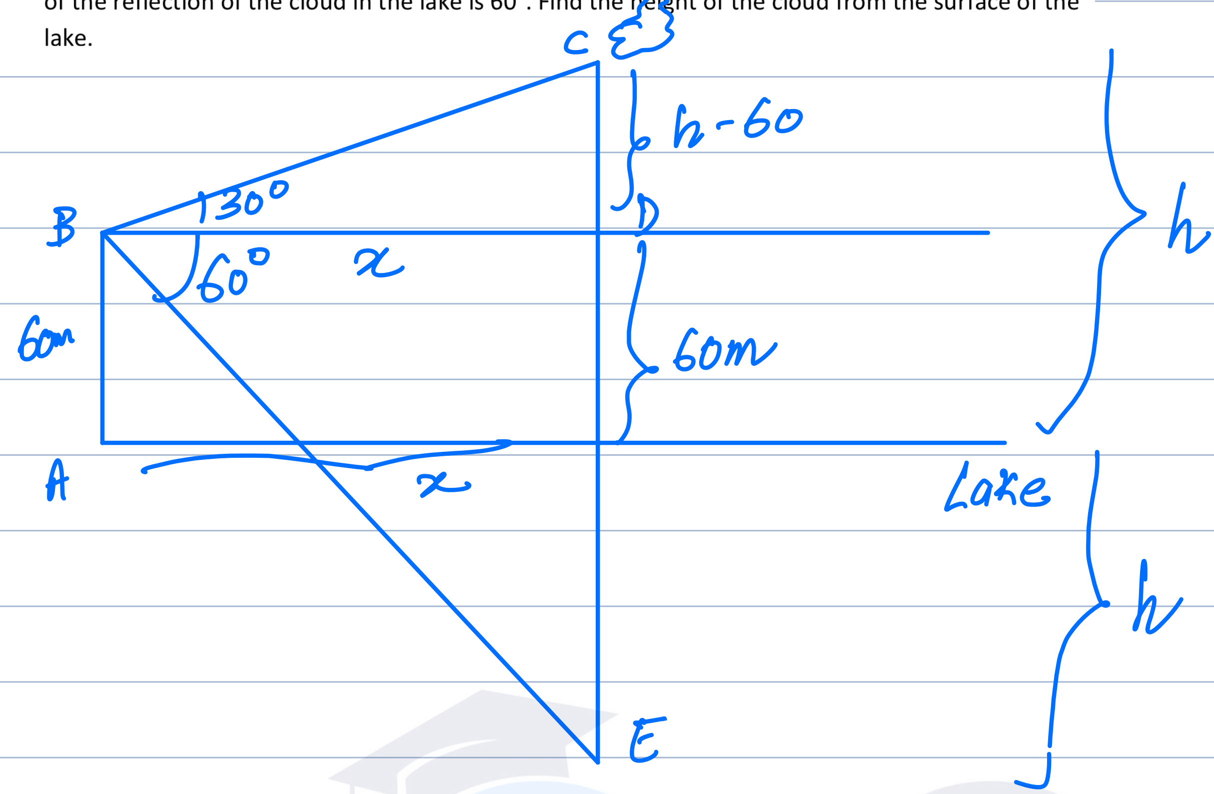
In $\triangle ACF_2$; $\tan 30° = \frac{h}{x+3000} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+3000} \Rightarrow x+3000 = \sqrt{3}h$

$\Rightarrow x + 3000 = \sqrt{3}(\sqrt{3}x)$

$\Rightarrow 3000 = 2x \Rightarrow x = 1500\,m. \Rightarrow h = 1500\sqrt{3}\,m.$

$h = 2598\,m$ Ans

12. The angle of elevation of a cloud from a point 60m above a lake is 30° and the angle of depression of the reflection of the cloud in the lake is 60°. Find the height of the cloud from the surface of the lake.



In $\triangle BCD$; $\tan 30^\circ = \frac{h-60}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h-60}{x}$

$\Rightarrow x = \sqrt{3}(h-60)$

In $\triangle BDE$; $\tan 60^\circ = \frac{h+60}{x} \Rightarrow \sqrt{3} = \frac{h+60}{x} \Rightarrow h+60 = \sqrt{3}x$.