

Recall:Linear equations →

$$2x + 3 = 0 \rightarrow x = -3/2$$

$$7x - 14 = 0 \rightarrow x = 2$$

$$-3x + 9 = 0 \rightarrow x = 3$$

$$ax + b = 0 \quad (a \neq 0)$$

↳ General form of a linear equation in one variable

General form of Linear Inequalities

1) $ax + b < 0$

2) $ax + b \leq 0$

3) $ax + b > 0$

4) $ax + b \geq 0$

Mathematical Notations.1) Natural Numbers
 $\{1, 2, 3, 4, 5, \dots\}$ → $\mathbb{N}$ or N 2) whole Numbers
 $\{0, 1, 2, 3, 4, 5, \dots\}$ → $\mathbb{W}$ or W

"Zahlen" → German word
↳ means number.

3) Integers
 $\{0, \pm 1, \pm 2, \pm 3, \pm 4, \dots\}$ → $\mathbb{Z}$ or Z or I

4) Positive Integers $\longrightarrow \mathbb{Z}^+$ or $\mathbb{Z}^+$ or $\mathbb{I}^+$
 $\{1, 2, 3, \dots\}$

5) Negative Integers $\longrightarrow \mathbb{Z}^-$ or $\mathbb{Z}^-$ or $\mathbb{I}^-$
 $\{-1, -2, -3, -4, \dots\}$
"Quoziente" $\rightarrow$ Italia word
 $\searrow$
Quotient.

6) Rational numbers $\longrightarrow \mathbb{Q}$ or $\mathbb{Q}$
 $\left\{ \frac{a}{b}; b \neq 0, a, b \text{ are Integers} \right\}$

7) Irrational numbers $\longrightarrow \mathbb{Q}'$ or $\mathbb{Q}'$

8) Real numbers $\longrightarrow \mathbb{R}$ or $\mathbb{R}$

$\in$ $\rightarrow$ belongs to

$\forall$ $\rightarrow$ for all or for every

$\cup$ $\rightarrow$ Union

$\cap$ $\rightarrow$ Intersection

Ex: 1) Let $A = \{x \mid x \in \mathbb{N} \ \& \ x \leq 5\}$

$$A = \{1, 2, 3, 4, 5\}$$

1 or;
↓
Such that

2) Let $A = \{x : x \in \mathbb{N} \ \& \ x < 5\}$

$$A = \{1, 2, 3, 4\}$$

Properties of Inequalities.

1) $a < b \Rightarrow a + k < b + k$

2) $a < b \Rightarrow a - k < b - k$

3) $a < b \Rightarrow ak < bk ; \quad k > 0$

4) $a < b \Rightarrow ak > bk ; \quad k < 0$

→ k is a Constant.

$$k \in \mathbb{R}$$

eg:- $2 < 5 \rightarrow$ Add 4 on both sides

$$\begin{array}{l} +4 \curvearrowright \\ 6 < 9 \end{array}$$

$2 < 5 \rightarrow$ Subtract 4 on both sides

$$\begin{array}{l} -4 \curvearrowright \\ -2 < 1 \end{array}$$

$$\begin{array}{l|l} \begin{array}{l} \times 3 \curvearrowright \\ 5 > 2 \\ 15 > 6 \end{array} & \begin{array}{l} \times -3 \curvearrowright \\ 5 > 2 \\ -15 < -6 \end{array} \end{array}$$

$$\begin{array}{l} 100 > 50 \\ \div 10 \curvearrowright \\ 10 > 5 \end{array}$$

$$\begin{array}{l} 100 > 50 \\ \div -10 \curvearrowright \\ -10 < -5 \end{array}$$

$$5) \quad a < b \Rightarrow \frac{a}{k} < \frac{b}{k} \quad (k > 0)$$

$$6) \quad a < b \Rightarrow \frac{a}{k} > \frac{b}{k} \quad (k < 0)$$

Lecture-II

Examples.

Solve the inequation & Represent the solution set on the number line.

$$1) \quad 12 + \frac{5}{6}x \leq 5 + 3x; \quad x \in \mathbb{R}$$

Solⁿ:

$$12 + \frac{11}{6}x \leq 5 + 3x$$

multiply both sides by 6.

$$\Rightarrow 72 + 11x \leq 30 + 18x$$

$$\Rightarrow 72 - 30 \leq 18x - 11x \Rightarrow 42 \leq 7x$$

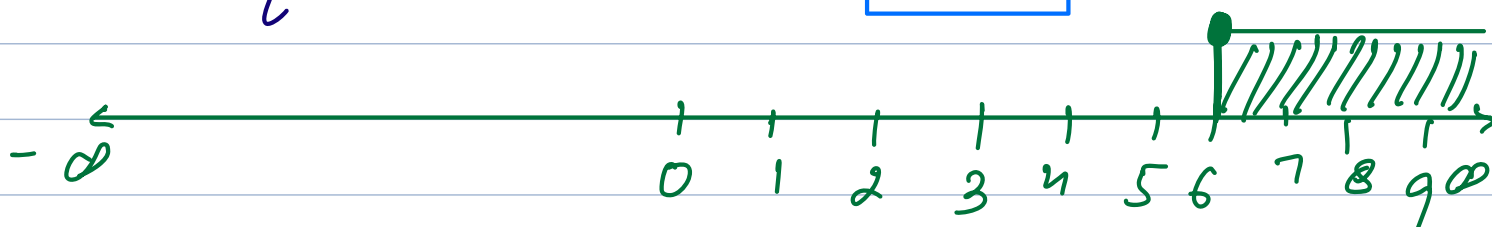
$$\Rightarrow \boxed{6 \leq x}$$

Divide both sides by 7

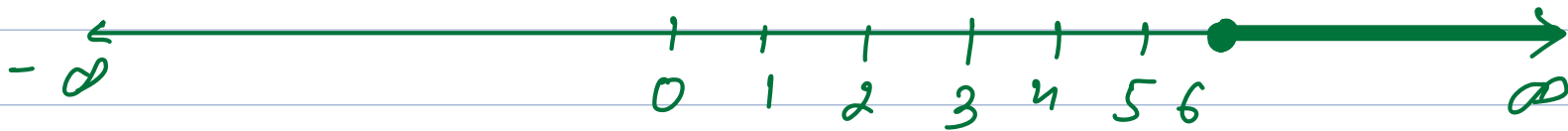
or

$$\boxed{x \geq 6}$$

$$\text{Solution Set} = \{ x \geq 6 : x \in \mathbb{R} \}$$



Q81



$$2) \quad 2x + \frac{7}{2} > \frac{5x}{3} + 3, \quad x \in \mathbb{I}$$

Sol:

$$2x + \frac{7}{2} > \frac{5x}{3} + 3$$

multiply by 6

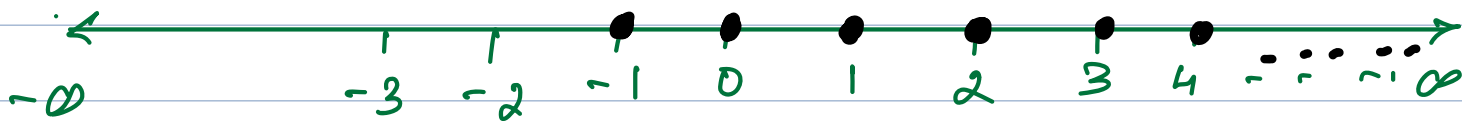
$$12x + 21 > 10x + 18$$

$$\Rightarrow 12x - 10x > 18 - 21 \Rightarrow 2x > -3$$

$$\Rightarrow x > \frac{-3}{2} \quad (\text{Ans})$$

$$\Rightarrow x > -1.5$$

Solution Set := $\{-1, 0, 1, 2, 3, \dots\}$ or $\{x > -1.5 : x \in \mathbb{I}\}$



$$3) \quad \frac{5x-8}{3} \geq \frac{4x-7}{2}, \quad x \in \mathbb{R}$$

Solⁿ:

$$\frac{5x-8}{3} \geq \frac{4x-7}{2}$$

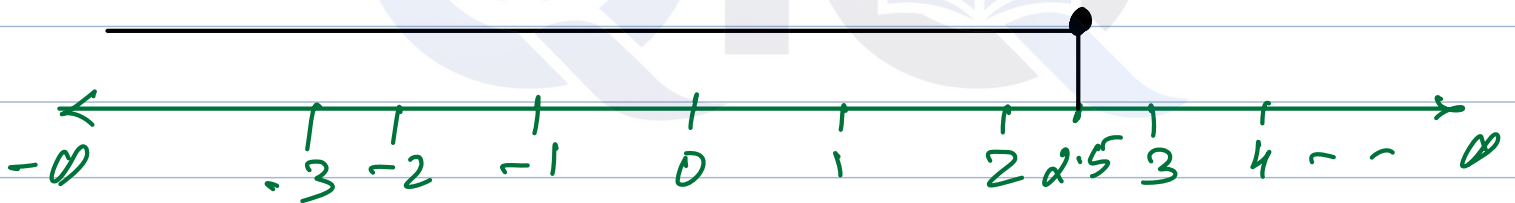
↪ Multiply by 6

$$2(5x-8) \geq 3(4x-7) \Rightarrow 10x-16 \geq 12x-21$$

$$\Rightarrow 21-16 \geq 12x-10x \Rightarrow 5 \geq 2x \Rightarrow \frac{5}{2} \geq x$$

$$\Rightarrow x \leq \frac{5}{2} \Rightarrow \boxed{x \leq 2.5}$$

Solution Set :- $\{x \leq 2.5 : x \in \mathbb{R}\}$



4) $2x-5 \leq 5x+4 < 11$, $x \in \mathbb{I}$

[2011]

Solⁿ: * $2x-5 \leq 5x+4 \Rightarrow -5-4 \leq 5x-2x \Rightarrow -9 \leq 3x$
 $\Rightarrow \boxed{-3 \leq x}$

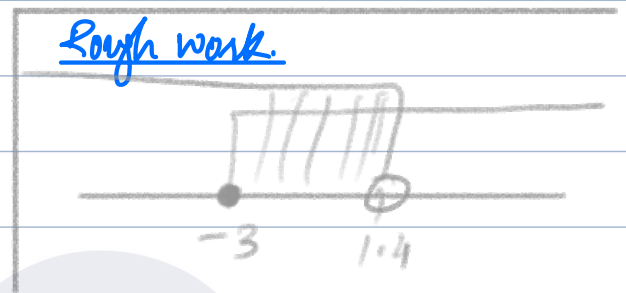
** $5x+4 < 11 \Rightarrow 5x < 11-4 \Rightarrow 5x < 7 \Rightarrow x < \frac{7}{5}$

$\Rightarrow \boxed{x < 1.4}$

$\Rightarrow -3 \leq x < 1.4$

$x = \{-3, -2, -1, 0, 1\}$

Ans.



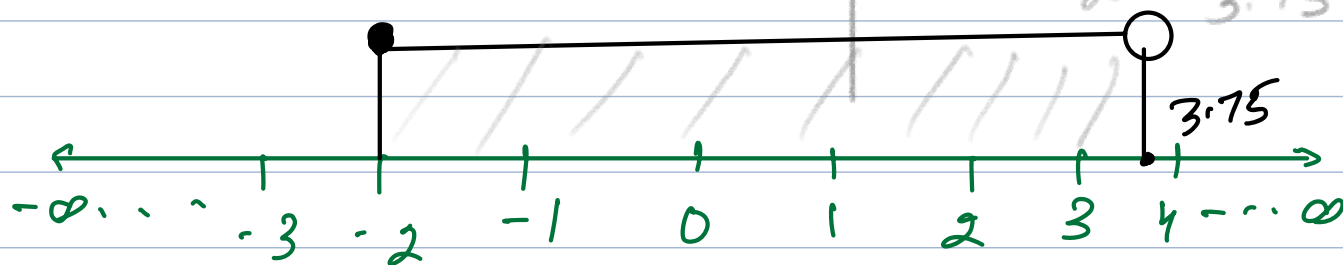
5) $-3 < \frac{-1}{2} - \frac{2x}{3} \leq \frac{5}{6}$, $x \in \mathbb{R}$.

Solⁿ: $-3 < \frac{-1}{2} - \frac{2x}{3} \leq \frac{5}{6} \xrightarrow{*6} -18 < -3-4x \leq 5$

* $-18 < -3-4x \Rightarrow 4x < -3+18 \Rightarrow 4x < 15 \Rightarrow x < \frac{15}{4}$
 $\Rightarrow \boxed{x < 3.75}$

** $-3-4x \leq 5 \Rightarrow -3-5 \leq 4x \Rightarrow -8 \leq 4x \Rightarrow \boxed{-2 \leq x}$

$$\{ -2 \leq x < 3.75 \mid x \in \mathbb{R} \}$$

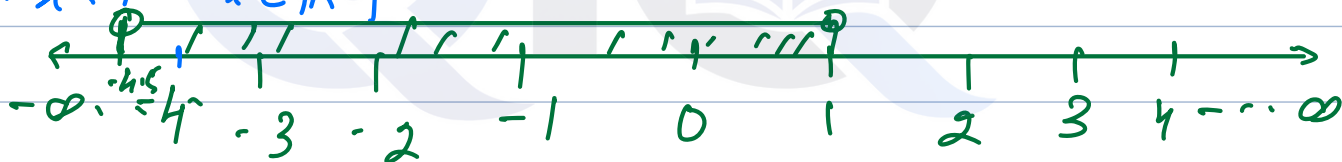


6) $13x - 5 < 15x + 4 < 7x + 12 \quad x \in \mathbb{R}$

Solⁿ: $\Rightarrow 13x - 5 < 15x + 4 \Rightarrow -5 - 4 < 15x - 13x \Rightarrow -9 < 2x$
 $\Rightarrow \boxed{-4.5 < x}$

$\Rightarrow 15x + 4 < 7x + 12 \Rightarrow 15x - 7x < 12 - 4 \Rightarrow 8x < 8$
 $\Rightarrow \boxed{x < 1}$

$\{ -4.5 < x < 1 \mid x \in \mathbb{R} \}$



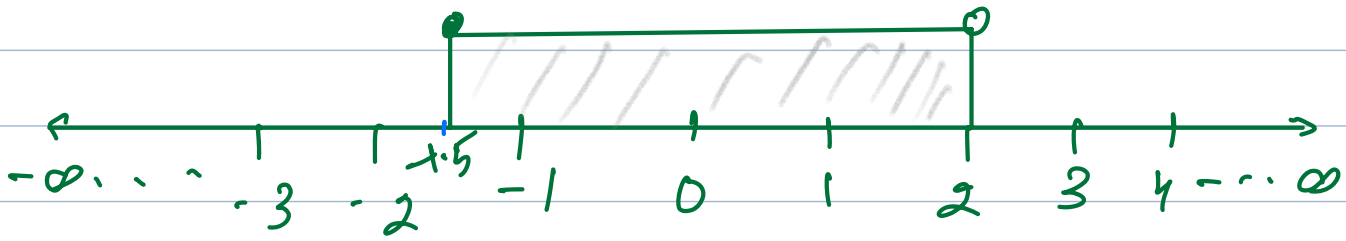
7) $-3(x-7) \geq 15-7x > \frac{x+1}{3} \quad x \in \mathbb{R}$

Solⁿ: $\Rightarrow -9(x-7) \geq 45-21x > x+1$

$\Rightarrow -9x + 63 \geq 45 - 21x \Rightarrow 21x - 9x \geq 45 - 63 \Rightarrow 12x \geq -18$
 $\Rightarrow x \geq \frac{-18}{12} \Rightarrow \boxed{x \geq -1.5}$

$\Rightarrow 45 - 21x > x + 1 \Rightarrow 45 - 1 > x + 21x \Rightarrow 44 > 22x$
 $\Rightarrow \boxed{2 > x}$

$$-1.5 \leq x < 2 \text{ \& } x \in \mathbb{R}$$



25. $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W$

$$\Rightarrow -\frac{17}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2 \xrightarrow{\times 6} -17 < 3 - 4x \leq 12$$

$$* -17 < 3 - 4x \Rightarrow 4x < 3 + 17 \Rightarrow 4x < 20 \Rightarrow \boxed{x < 5}$$

$$* 3 - 4x \leq 12 \Rightarrow 3 - 12 \leq 4x \Rightarrow -9 \leq 4x \Rightarrow -\frac{9}{4} \leq x$$

$$\Rightarrow \boxed{x \geq -2.25}$$

$$\{x: -2.25 \leq x < 5; x \in W\} = \{0, 1, 2, 3, 4\} \underline{\underline{Ans}}$$

29. $-2 + 10x \leq 13x + 10 < 24 + 10x, x \in Z$ (2018)

Sol: $* -2 + 10x \leq 13x + 10 \Rightarrow -2 - 10 \leq 13x - 10x \Rightarrow -12 \leq 3x$

$$\Rightarrow \boxed{-4 \leq x}$$

$$* 13x + 10 < 24 + 10x \Rightarrow 13x - 10x < 24 - 10 \Rightarrow 3x < 14$$

$$\Rightarrow x < \frac{14}{3} \Rightarrow \boxed{x < 4.67}$$

$$\{x: -4 \leq x < 4.67; x \in Z\} = \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$$