Probability

Total lecture = 1

Lecture-I

1) Sample space (S) → All the outcomes of the random experiment.

2) Events (E) → outcomes associated with sample space.

$$\boxed{P(E) = \frac{n(E)}{n(S)} = \frac{\text{no. of favorable outcomes}}{\text{no. of total outcomes.}}}$$

e.g:- Tossing a fair coin → $S = \{H, T\}$

Tossing two fair coins Simultaneously → $S = \{HH, HT, TH, TT\}$

Tossing three fair coins Simultaneously → $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

Illustration

1) Two coins are tossed together. Find the probability of getting
   i) 2 heads
   ii) atleast 1 tail

Solⁿ:- ∵ Two coins are tossed together

∴ Sample Space $S = \{HH, HT, TH, TT\} \Rightarrow n(S) = 4$

i) Let E_1 be the event of getting 2 heads

∴ $E_1 = \{HH\} \longrightarrow n(E_1) = 1$

$\because P(E_1) = \frac{n(E_1)}{n(S)} = 1/4$ Ans

ii) Let E_2 be the event of getting atleast 1 tail.
$E_2 = \{HT, TH, TT\}$

either 1 or more than 1

$\because P(E_2) = \frac{n(E_2)}{n(S)} = 3/4$ Ans

<u>Note</u>: If n coins are tossed simultaneously (together) then the number of total outcomes in the sample space is 2^n.

2) A die is thrown once. What is the probability of getting i) an even number
ii) a number greater than 2.

3) Cards marked with numbers 1, 2, 3, 4, ... 20 are well shuffled & a card is drawn at random. What is the probability that the number on the card is:

i) a prime number
ii) a number divisible by 3
iii) a perfect square number.

Ans: i) $2/5$ $\rightarrow E_1 = \{2, 3, 5, 7, 11, 13, 17, 19\} \Rightarrow P(E_1) = 8/20 = 2/5$.

ii) $3/10$ $\rightarrow E_2 = \{3, 6, 9, 12, 15, 18\} \rightarrow P(E_2) = 6/20 = 3/10$

iii) $1/5$ $\rightarrow E_3 = \{1, 4, 9, 16\} \Rightarrow P(E_3) = 4/20 = 1/5$.

4) A game of numbers has cards marked 11, 12, 13, ...40. A card is drawn at random. Find the probability that the card is

i) a perfect square

ii) divisible by 7.

Solⁿ: 4) $n(S) = 30$

i) $E_1 = \{16, 25, 36\} \Rightarrow P(E_1) = 3/30 = 1/10$.

ii) $E_2 = \{14, 21, 28, 35\} \Rightarrow P(E_2) = \frac{4}{30} = \frac{2}{15}$.

5) A box contains some black balls & 30 white balls. If the probability of drawing a black ball is two-fifth of that of drawing a white ball, find the number of black balls in the box.

Sol:- Let the number of black balls be x.

x Black balls
30 white balls

$n(S) = x+30$

Given that:- $P(B) = \frac{2}{5} P(W)$

$\Rightarrow \frac{x}{x+30} = \frac{2}{5} \times \frac{30}{x+30} \Rightarrow x = \frac{2}{5} \times 30 \Rightarrow \boxed{x=12}$ Ans

6) A bag contains 5 black balls, 7 blue balls, 4 white balls & 6 red balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is
i) Red ii) black or white iii) not blue iv) neither black nor blue

i) $P(R) = \frac{6}{22} = \frac{3}{11}$ ii) $P(B \text{ or } W) = \frac{9}{22}$

iii) $P(\text{Not Blue}) = \frac{15}{22}$ iv) $P(\text{Neither black nor Blue}) = \frac{10}{22} = \frac{5}{11}$.

7) Find the probability that leap year selected at random will contain 53 Sundays.

366 days

52 weeks 2 days.

$S = \{MT, TW, WTh, ThF, SatS, SM\}$

$E = \{SatS, SM\}$ ⟹ $P(53 \text{ Sundays}) = 2/7$

8) Find the probability that a non-leap year selected at random will contain 53 Sundays.

365 days

52 weeks, 1 day

$S = \{M, T, W, Th, F, Sat, S\}$

$E = \{S\}$ $P(53 \text{ Sundays}) = 1/7$.