Quadratic equations.

Total lecture = 3.

Lecture-I

Recall:-

An expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

is said to polynomial.

Polynomial of degree n.

where $n \in W$, $a_n \neq 0$

$a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0$ are Constat.

$n=0 \longrightarrow a_0 \longrightarrow$ Constant polynomial $(a \neq 0)$

$n=1 \longrightarrow a_1 x + a_0 \longrightarrow$ Linear polynomial $\rightarrow ax+b$.

$n=2 \longrightarrow a_2 x^2 + a_1 x + a_0 \longrightarrow$ Quadratic polynomial $\rightarrow ax^2+bx+c$

$n=3 \longrightarrow a_3 x^3 + a_2 x^2 + a_1 x + a_0 \longrightarrow$ Cubic polynomial $\rightarrow ax^3+bx^2+cx+d$

Quadratic equations

$$ax^2 + bx + c = 0 \quad (a \neq 0)$$

$$a, b, c \in \mathbb{R}$$

1) Coefficient of $x^2 = a$

Coefficient of $x = b$

Constant term $= c$

Ex: (a) $x^2+5x+6=0 \rightarrow$ Quadratic equation

Coefficient of $x^2 = 1$
Coefficient of $x = 5$
Constant term $= 6$

(b) $2x^2-9x-5=0$

Coefficient of $x^2 = 2$
Coefficient of $x = -9$
Constant term $= -5$

→ Roots 'or' zeroes 'or' solutions

Roots of a Quadratic Equations.

* → Completing the square Method
# → Quadratic formula.
# → Splitting the Middle Term
or
Middle term factorisation

Explain :

$$ax^2+bx+c=0 \qquad (a\neq 0)$$

$$a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right)=0 \Rightarrow x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\Rightarrow x^2+2\cdot\frac{b}{2a}x+\frac{c}{a}=0$$

$$\Rightarrow \underbrace{x^2+2\cdot\frac{b}{2a}x+\left(\frac{b}{2a}\right)^2}-\left(\frac{b}{2a}\right)^2+\frac{c}{a}=0$$

$$\Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a} \Rightarrow \left(x+\frac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\Rightarrow x+\frac{b}{2a}=\pm\sqrt{\frac{b^2-4ac}{4a^2}} \Rightarrow x+\frac{b}{2a}=\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow \boxed{x=\frac{-b\pm\sqrt{D}}{2a}}$$

where $D=b^2-4ac$

Discriminant

Quadratic formula

Or,

Shridharacharya formula.

Find the roots of the given below equations.

1) $x^2+5x+6=0$

Sol^n: $a=1$, $b=5$, $c=6$

$D = b^2-4ac = (5)^2-4(1)(6) = 25-24 = 1$

$\because x = \dfrac{-b\pm\sqrt{D}}{2a}$ $\therefore x = \dfrac{-5\pm\sqrt{1}}{2(1)} = \dfrac{-5\pm1}{2}$

(+) $x = \dfrac{-5+1}{2} = -2 \Rightarrow \boxed{x=-2}$

'or'

(−) $x = \dfrac{-5-1}{2} = -3 \Rightarrow \boxed{x=-3}$

Two roots

2) $x^2-18x+81=0$

Sol^n: Here; $a=1$, $b=-18$, $c=81$, $D = b^2-4ac = (-18)^2-4(1)(81)$

$= 324-324$

$D=0$

$\because x = \dfrac{-b\pm\sqrt{D}}{2a}$ $\therefore x = \dfrac{-(-18)\pm\sqrt{0}}{2(1)}$

$\Rightarrow x = \dfrac{18}{2} \Rightarrow \boxed{x=9}$

1 root.

3) $x^2+x+1=0$

Here; $a=b=c=1$; $D = b^2-4ac = (1)^2-4(1)(1) = 1-4 = -3$

$\boxed{D=-3}$

$\because x = \dfrac{-b\pm\sqrt{D}}{2a}$

$\therefore x = \dfrac{-(1)\pm\sqrt{-3}}{2(1)}$

$\sqrt{-3}$ is not a real number.

or

$\sqrt{-3}$ is an imaginary no.

No root

$\sqrt{\text{negative number}}$ is an imaginary no.

$\boxed{\text{Lecture-II}}$

Nature of roots.

1) $D > 0$ $\longrightarrow$ Real & Distinct roots
(Two real roots)

2) $D = 0$ $\longrightarrow$ Real & equal roots.
(one real root)

3) $D < 0$ $\longrightarrow$ No real roots
or
Imaginary roots or Complex roots

1) Without solving the following equations, find the value of p for which the roots are real & equal.

(a) $px^2 - 4x + 3 = 0 \longrightarrow a = p,\ b = -4,\ c = 3$

Sol^n:

$\because$ Roots are real & equal

$\therefore$ $D = 0 \Rightarrow b^2 - 4ac = 0$

$\Rightarrow (-4)^2 - 4(p)(3) = 0$

$16 - 12p = 0 \Rightarrow p = \frac{16}{12} \Rightarrow \boxed{p = \frac{4}{3}}$

Ans

(b) $x^2 + (p-3)x + p = 0 \longrightarrow a = 1 \quad b = p-3$

$c = p$

Sol^n:

$\because$ Roots are real & equal

$\therefore$ $D = 0 \Rightarrow b^2 - 4ac = 0 \Rightarrow (p-3)^2 - 4(1)(p) = 0$

$\Rightarrow p^2 - 6p + 9 - 4p = 0 \Rightarrow p^2 - 10p + 9 = 0$

$\Rightarrow p^2 - 9p - p + 9 = 0 \Rightarrow p(p-9) - 1(p-9) = 0$

$\Rightarrow (p-9)(p-1) = 0 \Rightarrow p = 9$ OR $p = 1$.

2) Find the value(s) of k for which

$$Kx^2 + Kx + 1 = -4x^2 - x$$ has equal roots

Solⁿ: $Kx^2 + 4x^2 + Kx + x + 1 = 0 \Rightarrow (K+4)x^2 + (K+1)x + 1 = 0$

Here; $a = K+4$, $b = K+1$, $c = 1$

$\because$ Roots are real & equal

$\therefore D = 0 \Rightarrow b^2 - 4ac = 0$

$\Rightarrow (K+1)^2 - 4(K+4)(1) = 0$

$\Rightarrow K^2 + 2K + 1 - 4K - 16 = 0$

$\Rightarrow K^2 - 2K - 15 = 0$

$\Rightarrow K^2 - 5K + 3K - 15 = 0$

$\Rightarrow K(K-5) + 3(K-5) = 0$

$\Rightarrow (K-5)(K+3) = 0$

$\Rightarrow K = 5$ or $K = -3$ Ans

3) Solve for x & give your answer correct to 2 decimal places.

(a) $x^2 - 10x + 6 = 0$

Solⁿ: Here; $a = 1$, $b = -10$, $c = 6$

$D = b^2 - 4ac = (-10)^2 - 4(1)(6)$

$D = 100 - 24 = 76$

$\because x = \dfrac{-b \pm \sqrt{D}}{2a}$ $\therefore x = \dfrac{-(-10) \pm \sqrt{76}}{2(1)}$

$\Rightarrow x = \dfrac{10 + \sqrt{76}}{2}$ or $x = \dfrac{10 - \sqrt{76}}{2}$

$$\Rightarrow \quad x = \frac{10 + 8.718}{2} \quad \text{or} \quad x = \frac{10 - 8.718}{2}$$

$$x = \frac{18.718}{2} \quad \text{or} \quad x = \frac{1.282}{2}$$

$$x = 9.36 \quad \text{or} \quad x = 0.64 \quad \underline{\underline{\text{Ans}}}$$

(b) $x^2 + 7x = 7 \longrightarrow x^2 + 7x - 7 = 0$

Solⁿ: Here; $a = 1$, $b = 7$, $c = -7$ $\qquad \{\sqrt{77} = 8.775\}$

$\therefore D = b^2 - 4ac = (7)^2 - 4(1)(-7) = 49 + 28 = 77$

$$\because \quad x = \frac{-b \pm \sqrt{D}}{2a} \quad \therefore \quad x = \frac{-7 \pm \sqrt{77}}{2(1)}$$

$$\Rightarrow \quad x = \frac{-7 \pm 8.775}{2}$$

$$\Rightarrow \quad x = \frac{-7 + 8.775}{2} \quad \text{or} \quad x = \frac{-7 - 8.775}{2}$$

$$x = \frac{1.775}{2} \quad \text{or} \quad x = \frac{-15.775}{2}$$

$$x = 0.89 \quad \text{or} \quad x = -7.89 \quad \underline{\underline{\text{Ans}}}$$

(c) $x^2 - 4x - 8 = 0$ $\qquad \sqrt{48} = 6.928.$

$\rightarrow x = 5.47$ or $x = -1.47$

(d) $2x^2 + \sqrt{5}x - 5 = 0$ $\qquad \sqrt{5} = 2.23$

$\rightarrow x = -2.24$ or $x = 1.12$ $\qquad \sqrt{45} = 6.708.$

Soln. $a = 2, \quad b = \sqrt{5}, \quad c = -5, \quad D = b^2 - 4ac$

$$D = (\sqrt{5})^2 - 4(2)(-5) = 5 + 40 = 45$$

$\because x = \dfrac{-b \pm \sqrt{D}}{2a}$ $\quad \therefore x = \dfrac{-\sqrt{5} \pm \sqrt{45}}{2(2)}$

$x = \dfrac{-\sqrt{5} + \sqrt{45}}{4}$ or $x = \dfrac{-\sqrt{5} - \sqrt{45}}{4} \Rightarrow x = 1.12$ or $x = -2.24$ Ans

Lecture-II

(e) $\dfrac{x+3}{2x+3} = \dfrac{x+1}{3x+2}$

Sol:- $(x+3)(3x+2) = (x+1)(2x+3)$

$\Rightarrow 3x^2 + 2x + 9x + 6 = 2x^2 + 3x + 2x + 3$

$\Rightarrow 3x^2 + 11x + 6 = 2x^2 + 5x + 3 \Rightarrow x^2 + 6x + 3 = 0$

$D = b^2 - 4ac = (6)^2 - 4(1)(3) = 24$

$$x = \frac{-6 \pm \sqrt{24}}{2} \Rightarrow \frac{-6 + 4.899}{2} = -0.55 \text{ Ans}$$

$$\Rightarrow \frac{-6 - 4.899}{2} = -5.45 \text{ Ans}$$

Ex → 5 A

10) Solve:- $3x^2 = 5x \rightarrow 3x^2 - 5x = 0$

$$x(3x-5) = 0$$

$\Rightarrow$ Either $x = 0$ or $3x - 5 = 0$

$\hookrightarrow x = 0$ or $x = 5/3$ Ans

39) Solve:- $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

$$\Rightarrow \frac{(x-1) + 2(x-2)}{(x-2)(x-1)} = \frac{6}{x} \Rightarrow \frac{x-1+2x-4}{x^2-x-2x+2} = \frac{6}{x}$$

$$\Rightarrow \frac{3x-5}{x^2-3x+2} = \frac{6}{x} \Rightarrow 3x^2 - 5x = 6x^2 - 18x + 12$$

$$\Rightarrow 0 = 6x^2 - 3x^2 - 18x + 5x + 12$$

$$\Rightarrow 3x^2 - 13x + 12 = 0$$

$$\Rightarrow 3x^2 - 9x - 4x + 12 = 0$$

$$\Rightarrow 3x(x-3) - 4(x-3) = 0$$

$$\Rightarrow (x-3)(3x-4) = 0$$

$\Rightarrow x - 3 = 0$ or $3x - 4 = 0$

$\Rightarrow x = 3$ or $x = 4/3$

Ex:- (5C)

17) find K for which (a) $(K+1)x^2 + 2(K+3)x + K+8 = 0$ has equal roots.

Solⁿ:-

equal roots $\Rightarrow$ $D = 0$

$a = K+1$

$b = 2K+6$

$c = K+8$

$(2K+6)^2 - 4(K+1)(K+8) = 0$ $\rightarrow$ $b^2 - 4ac = 0$

$\Rightarrow (4K^2 + 36 + 24K) - 4(K^2 + 9K + 8) = 0$

$\Rightarrow \cancel{4K^2} + 24K + 36 - \cancel{4K^2} - 36K - 32 = 0$

$\Rightarrow -12K + 4 = 0 \Rightarrow 12K = 4 \Rightarrow \boxed{K = 1/3}$

(b) $3Kx^2 = 4(Kx - 1)$

$\Rightarrow 3Kx^2 = 4Kx - 4$

$\Rightarrow 3Kx^2 - 4Kx + 4 = 0$

$a = 3K$

$b = -4K$

$c = 4$

$D = 0 \Rightarrow (-4K)^2 - 4(3K)(4) = 0$

$\Rightarrow 16K^2 - 48K = 0$

$\Rightarrow 16K(K-3) = 0$

$\Rightarrow \cancel{K=0}$ or $K = 3$ Ans

23) $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ has equal roots.

Sol:

$D=0 \Rightarrow [-2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)=0$

$\Rightarrow 4(a^4-2a^2bc+b^2c^2)-4(b^2c^2-ac^3-ab^3+a^2bc)=0$

$a^4-2a^2bc+\cancel{b^2c^2}-\cancel{b^2c^2}+ac^3+ab^3-a^2bc=0$

$\Rightarrow a^4+ab^3+ac^3-3a^2bc=0$

$\Rightarrow a(a^3+b^3+c^3-3abc)=0 \Rightarrow a=0$ or

$a^3+b^3+c^3=3abc$