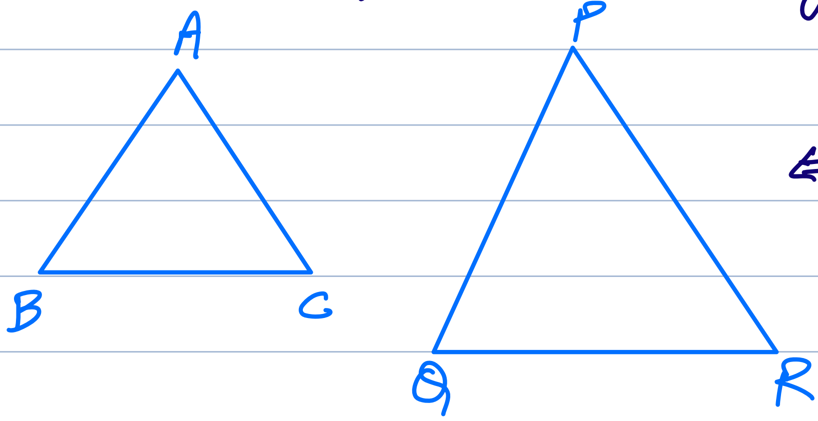
Total lecture = 3.

Lecture-I

Similar Triangles.

Two triangles are said to be Similar if & only if their all angles are equal & Corresponding sides are proportional.



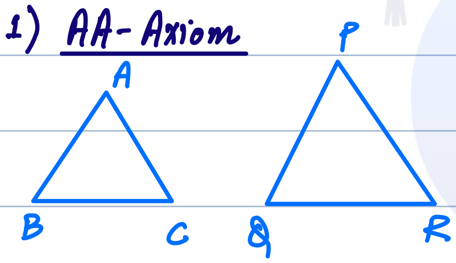
$\triangle ABC \sim \triangle PQR$

$\Leftrightarrow \angle A = \angle P,\ \angle B = \angle Q,\ \angle C = \angle R$

& $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$.

'$\sim$' read as "is Similar to"

1) AA- Axiom

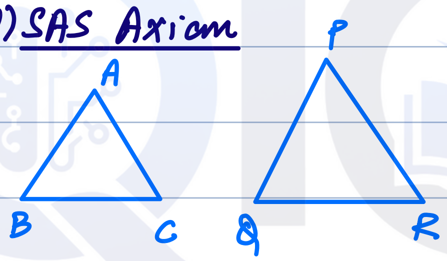


If $\angle A = \angle P$ & $\angle B = \angle Q$

then $\triangle ABC \sim \triangle PQR$

(by AA- Axiom)

2) SAS Axiom

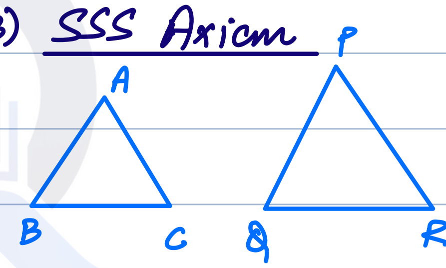


If $\dfrac{AB}{PQ} = \dfrac{BC}{QR}$ & $\angle B = \angle Q$

then $\triangle ABC \sim \triangle PQR$

(By SAS- Axiom)

3) SSS Axiom



If $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$ then

$\triangle ABC \sim \triangle PQR$ (by SSS Axiom).

Ex: $\triangle PRQ \sim \triangle LNM$

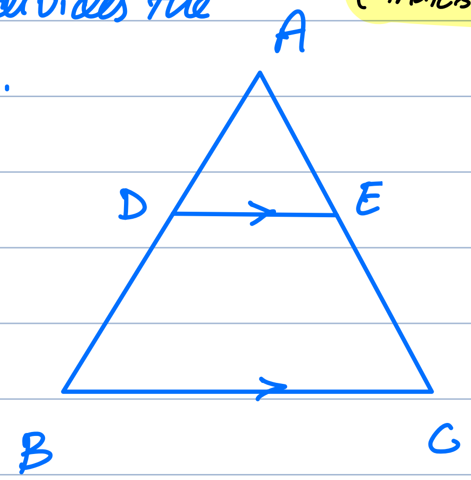
$\angle P = \angle L,\ \angle R = \angle N,\ \angle Q = \angle M$

$\dfrac{PR}{LN} = \dfrac{RQ}{NM} = \dfrac{PQ}{LM}$.

Basic Proportionality Theorem (BPT)

(Thale's Theorem)

Any line parallel to one side of a triangle divides the other two sides in same proportion.



Given:- $DE \parallel BC$

To Prove:- $\frac{AD}{DB} = \frac{AE}{EC}$.

Pf: In $\triangle ADE$ & $\triangle ABC$

$\angle DAE = \angle BAC$ (Common)

$\angle ADE = \angle ABC$ (Corresponding angles)

$\Rightarrow \triangle ADE \sim \triangle ABC$ (by AA-Axiom).

$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$. (by cpst)

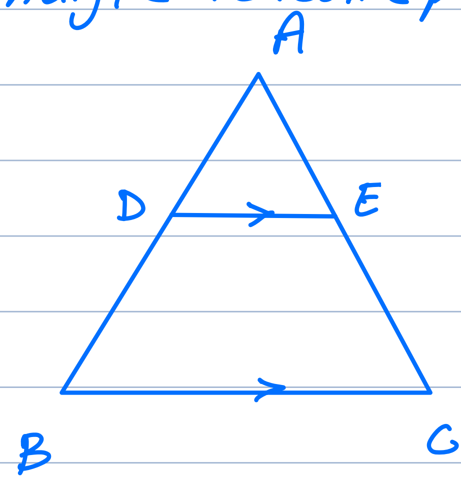
Now; $\frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$ (by taking reciprocal)

$\Rightarrow \frac{AB}{AD} - 1 = \frac{AC}{AE} - 1 \Rightarrow \frac{AB-AD}{AD} = \frac{AC-AE}{AE} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$

$\Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$ (by taking reciprocal)

Converse of Basic Proportionality Theorem:

If a line divides the two sides of a triangle in same proportion then it's parallel to third sides.



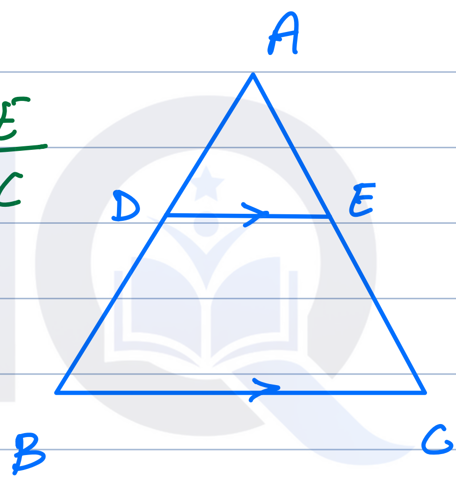
Given: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

Then $\Rightarrow$ $DE \parallel BC$

Remark.

1) $DE \parallel BC \Longleftrightarrow \dfrac{AD}{DB} = \dfrac{AE}{EC}$

(by BPT)



2) $\triangle ADE \sim \triangle ABC$

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}.$$

3) General Mistake $\rightarrow$ $\boxed{\dfrac{AD}{DB} = \dfrac{AE}{EC} = \dfrac{DE}{BC}}$

$\dfrac{AD}{DB} = \dfrac{AE}{EC} \neq \dfrac{DE}{BC}.$

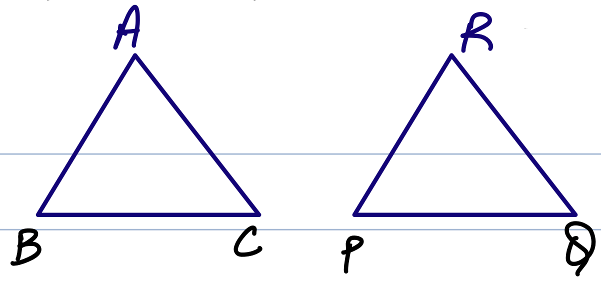
Lecture-II

Illustration.

$\frac{1}{2} = \frac{6}{QR}$

$QR = 12\,cm.$

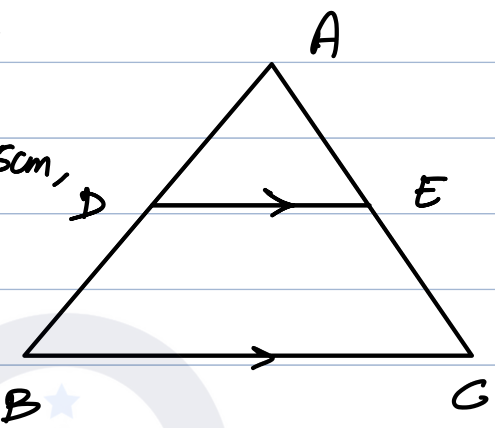
1. If $\Delta ABC \sim \Delta RPQ$, AB = 3 cm, BC = 5 cm, AC = 6 cm, RP = 6 cm and PQ = 10, then find QR.



$$\frac{AB}{RP} = \frac{BC}{PQ} = \frac{AC}{QR}$$

$$\frac{1}{2} = \frac{3}{6} = \frac{5}{10} = \frac{6}{QR}$$

2) In the adjoining figure $PE \parallel BC$.



i) if AD = 3.4 cm, AB = 8.5 cm & AC = 13.5 cm, find AE.

ii) if $\frac{AD}{DB} = \frac{3}{5}$ & AC = 9.6 cm find AE.

Soln i) In ΔADE & ΔABC; $\angle DAE = \angle BAC$ (Common)

& $\angle ADE = \angle ABC$ (Corresponding angles)

$\Rightarrow \Delta ADE \sim \Delta ABC$ (by AA Axiom)

$$\frac{AD}{AB} = \frac{AE}{AC} \Rightarrow \frac{3.4}{8.5} = \frac{AE}{13.5} \Rightarrow AE = \frac{3.4 \times 13.5}{8.5}$$

$$\Rightarrow AE = 5.4\,cm.$$

ii) $\because DE \parallel BC \quad \therefore \frac{AD}{DB} = \frac{AE}{EC}$

$$\Rightarrow \frac{3}{5} = \frac{AE}{EC} \Rightarrow \frac{3}{5} = \frac{x}{9.6-x}$$

Let $AE = x$ cm

$\Rightarrow EC = (9.6 - x)$ cm

$$\Rightarrow 3(9.6 - x) = 5x$$

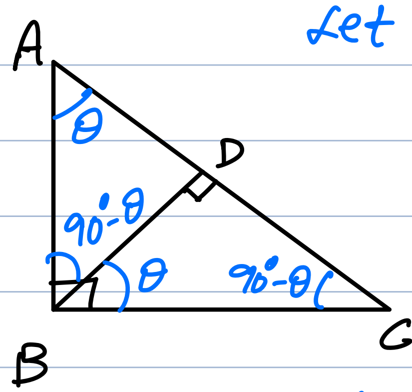
$$\Rightarrow x = 3.6\,cm = AE$$

3) In the given figure, $\triangle ABC$ is right-angled at B & $BD \perp AC$.

P.T i) $\triangle ADB \sim \triangle BDC$ ii) $\triangle ADB \sim \triangle ABC$

iii) $\triangle BDC \sim \triangle ABC$ iv) $BD^2 = AD \times DC$

Let $\angle BAD = \theta$.



i) In $\triangle ADB$ & $\triangle BDC$

& iv) $\angle ADB = \angle BDC$ $(=90°)$

$\angle ABD = \angle BCD$ (as shown in the figure)

$\Rightarrow$ $\triangle ADB \sim \triangle BDC$ (by AA-Axiom)

1 $\frac{AD}{BD} = \frac{BD}{CD}$ (by cpst) $\Rightarrow$ $AD \times CD = (BD)^2$.

ii) In $\triangle ADB$ & $\triangle ABC$ $\rightarrow$ $\angle ADB = \angle ABC$ $(=90°)$

$\angle DAB = \angle BAC$ (Common)

$\Rightarrow$ $\triangle ADB \sim \triangle ABC$ (by AA Axiom)

2

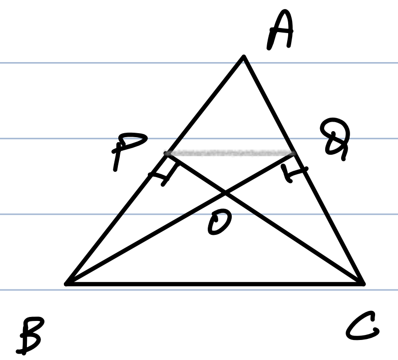
iii) from 1 & 2

$\triangle BDC \sim \triangle ABC$.

4) The altitude BQ & CP of $\triangle ABC$ meet at O. P.T

i) $CQ \cdot OP = BP \cdot OQ$

ii) $\triangle POQ \sim \triangle BOC$



we don't know whether $PQ \parallel BC$.

Pf: i) In $\triangle OPB$ & $\triangle OQC$

$\angle OPB = \angle OQC$ ($=90^\circ$; given)

$\angle POB = \angle QOC$ (vertically opp)

$\Rightarrow \triangle OPB \sim \triangle OQC$ (by A.A Axiom)

$\Rightarrow \underbrace{\frac{OP}{OQ} = \frac{PB}{QC}}$ $= \frac{OB}{OC}$ — (i)

$\hookrightarrow \frac{OP}{OQ} = \frac{BP}{CQ} \Rightarrow CQ \cdot OP = BP \cdot OQ.$ H.P.

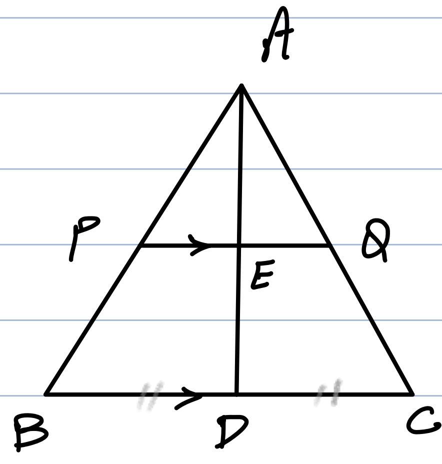
ii) In $\triangle POQ$ & $\triangle BOC$; $\angle POQ = \angle BOC$ (vert. opp.)

$\frac{OP}{OQ} = \frac{OB}{OC}$ (from (i))

$\Rightarrow \triangle POQ \sim \triangle BOC$ (by SAS Axiom).

(Imp. Q).

5) In the adjoining figure, $PQ \parallel BC$. Prove that median AD bisects PQ



To Prove: $PE = QE$.

Pf: In $\triangle APE$ & $\triangle ABD$

$\angle A = \angle A$ (Common)

$\angle APE = \angle ABD$ (Corresponding angles)

$\Rightarrow \triangle APE \sim \triangle ABD$ (by AA Axiom)

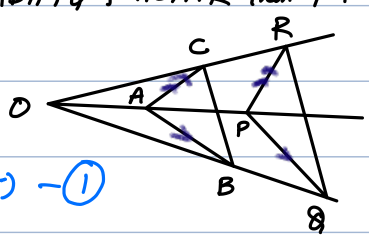
$\Rightarrow \dfrac{AP}{AB} = \dfrac{PE}{BD} = \dfrac{AE}{AD}$ — (1)

Similarly $\triangle AEQ \sim \triangle ADC \Rightarrow \dfrac{AE}{AD} = \dfrac{EQ}{DC} = \dfrac{AQ}{AC} \rightarrow$ (2)

from (1) & (2) $\dfrac{PE}{BD} = \dfrac{QE}{CD} \Rightarrow PE = QE$

{$\because BD = CD$ as AD is the median}

6) In the given figure $AB \parallel PQ$ & $AC \parallel PR$ Then P.T $BC \parallel QR$



Pf: In $\triangle OPR$; $AC \parallel PR$

$\Rightarrow \dfrac{OA}{AP} = \dfrac{OC}{CR}$ (by BPT) — (1)

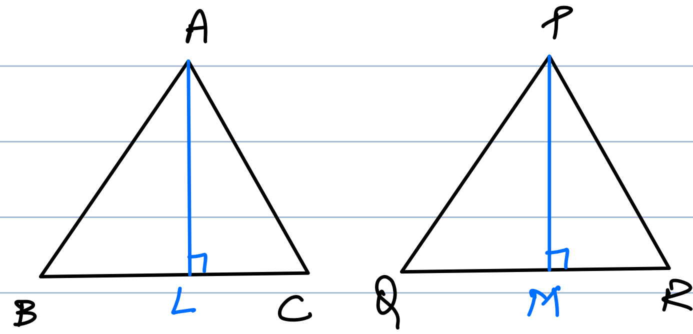
In $\triangle OPQ$; $AB \parallel PQ$

$\Rightarrow \dfrac{OA}{AP} = \dfrac{OB}{BQ}$ (by BPT) — (2) from (1) & (2) $\Rightarrow \dfrac{OC}{CR} = \dfrac{OB}{BQ}$.

Now; In $\triangle OQR \Rightarrow \dfrac{OC}{CR} = \dfrac{OB}{BQ}$. $\Rightarrow BC \parallel QR$ (by Converse of BPT).

Theorem

Lecture-III



If $\triangle ABC \sim \triangle PQR$

then

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}.$$

Pf: $\because \triangle ABC \sim \triangle PQR \quad \therefore \frac{AB}{PQ} = \frac{BC}{QR}$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{\cancel{\frac{1}{2}} \times BC \times AL}{\cancel{\frac{1}{2}} \times QR \times PM} = \frac{BC}{QR} \times \frac{AL}{PM}$$

In $\triangle ALB$ & $\triangle PMQ$; $\angle ABL = \angle PQM$ (similar $\triangle$)

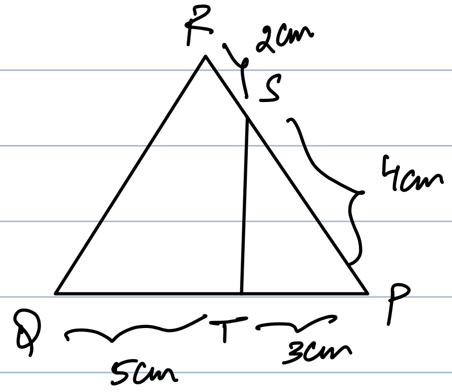
$\angle ALB = \angle PMQ$ $(= 90^\circ)$

$\Rightarrow \triangle ALB \sim \triangle PMQ$

$\Rightarrow \frac{AL}{PM} = \frac{AB}{PQ} \Rightarrow \frac{AL}{PM} = \frac{BC}{QR}$

$$\Rightarrow \frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{BC}{QR} \times \frac{BC}{QR} = \frac{(BC)^2}{(QR)^2}.$$

#



T.P.: $\Delta PQR \sim \Delta PST$

Pf: In ΔPQR & ΔPST

$\angle QPR = \angle SPT$ (Common)

& $\frac{PQ}{PS} = \frac{PR}{PT}$

$\frac{PQ}{PS} = \frac{8}{4} = 2$

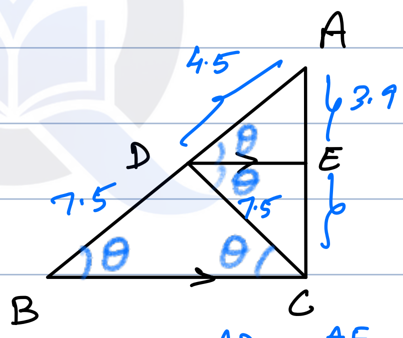
$\frac{PR}{PT} = \frac{6}{3} = 2$

$\Rightarrow \Delta PQR \sim \Delta PST$ (by SAS Axiom).

$\Rightarrow \frac{PQ}{PS} = \frac{PR}{PT} = \frac{QR}{ST}$

$2 = \frac{5.8}{ST} \Rightarrow ST = 2.9$ cm Ans

BD=DC & DE//BC.



T.P. $\angle ADE = \angle CDE$ (DE bisects $\angle ADC$)

Pf: $\angle ADE = \angle ABC$ (Corresponding angles)

$\angle ABC = \angle DCB$ ($\because BD = CD$)

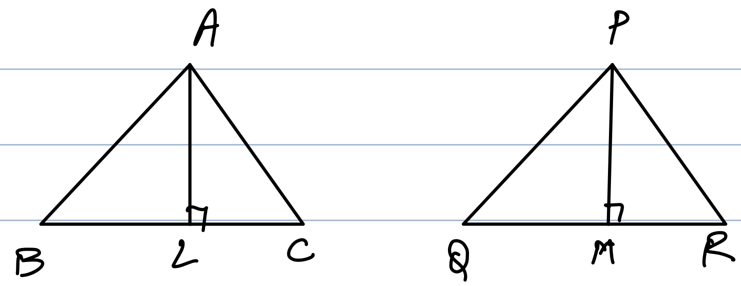
$\angle CDE = \angle DCB$ (Alternate interior)

$\angle ADE = \angle CDE$ (H.P.)

ii) $\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{4.5}{7.5} = \frac{3.9}{EC}$ ✓

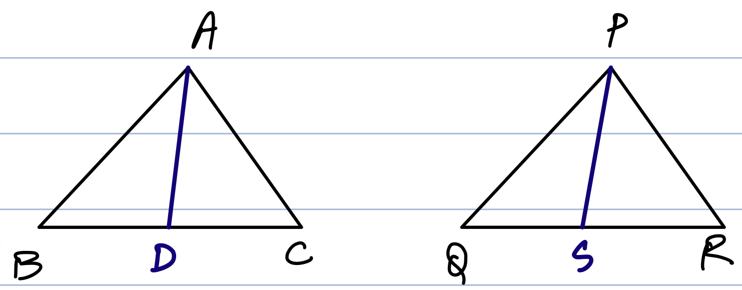
Theorem.



If $\triangle ABC \sim \triangle PQR$
then

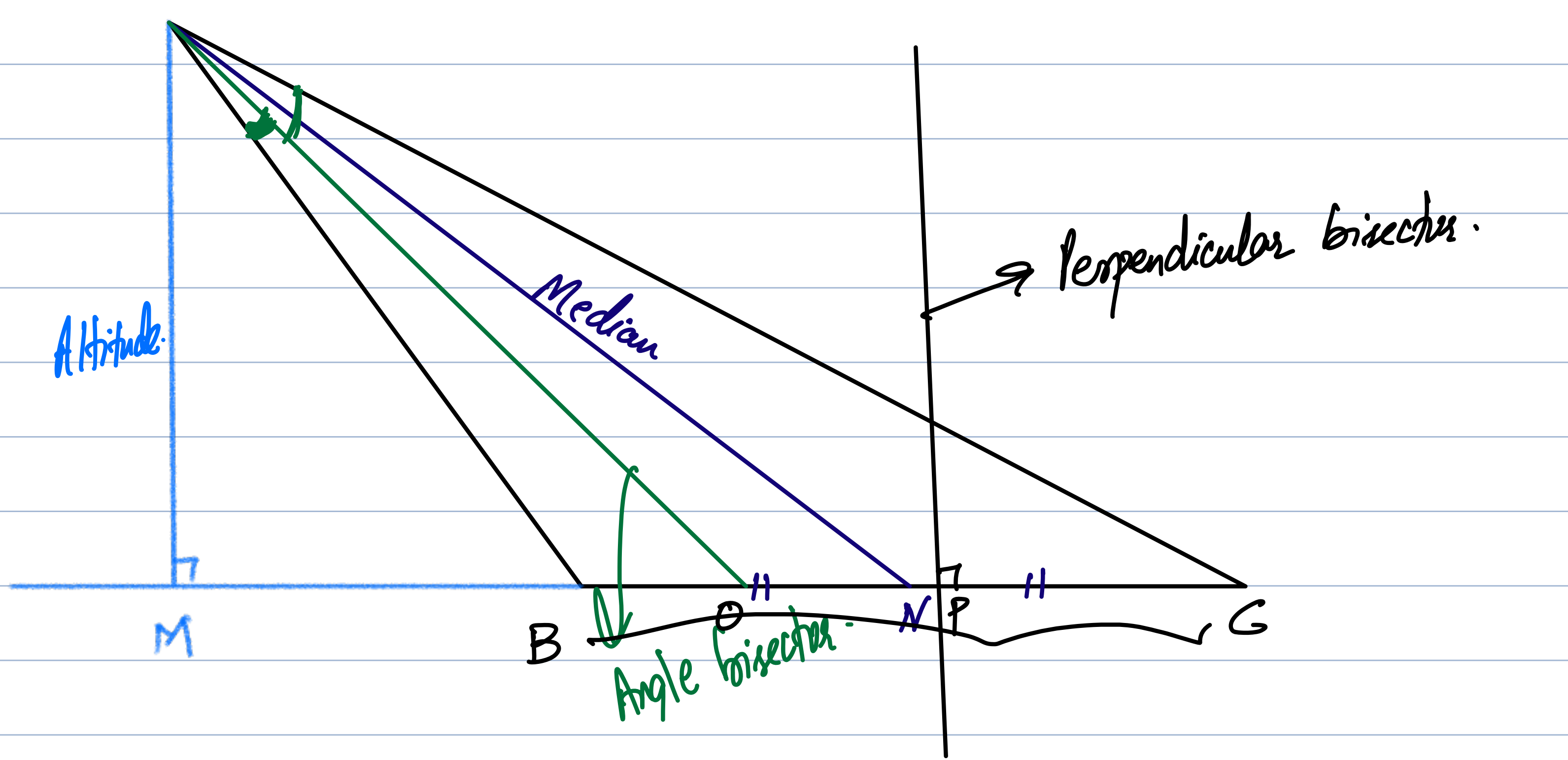
$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2} = \frac{(AL)^2}{(PM)^2}$$

Theorem AD & PS are Median.



$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle PQR} = \frac{(AD)^2}{(PS)^2}.$$

Explain the difference between Median, Altitude & perpendicular bisector.



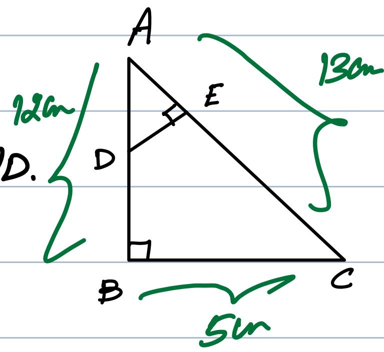
Exam style question.

1) ABC is a right angled triangle with $\angle ABC = 90°$. D is a point on AB & DE is perpendicular to AC. Prove that:

i) $\triangle ADE \sim \triangle ACB$

ii) If AC = 13cm, BC = 5cm & AE = 4cm, find DE & AD.

iii) find area of $\triangle ADE$: area of quad. BCED



Pf: i) In $\triangle ADE$ & $\triangle ACB$

$\angle DAE = \angle CAB$ (Common)

$\angle AED = \angle ABC$ ($= 90°$)

$\Rightarrow \triangle ADE \sim \triangle ACB$ (by AA-Axiom)

ii) $\Rightarrow \dfrac{AD}{AC} = \dfrac{DE}{CB} = \dfrac{AE}{AB}$ (by cpst)

$\Rightarrow \dfrac{AD}{13} = \dfrac{DE}{5} = \dfrac{4}{12}$

$\dfrac{DE}{5} = \dfrac{1}{3} \Rightarrow DE = \dfrac{5}{3}$ cm

$DE = 1.67$cm

$\dfrac{AD}{13} = \dfrac{1}{3} \Rightarrow AD = \dfrac{13}{3}$ cm

$AD = 4.33$cm

iii) $\because \triangle ADE \sim \triangle ACB$ $\therefore \dfrac{\text{area of } \triangle ADE}{\text{area of } \triangle ACB} = \left(\dfrac{AE}{AB}\right)^2 = \left(\dfrac{4}{12}\right)^2$

$\dfrac{\text{area of } \triangle ADE}{\text{area of } \triangle ACB} = \dfrac{1}{9}$

$\Rightarrow$ area of $\triangle ADE = K$ units^2

area of $\triangle ACB = 9K$ units^2.

$\Rightarrow$ area of BCED $= 8K$ units^2.

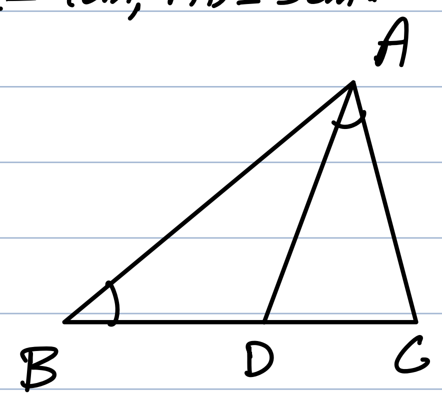
area of $\triangle ADE$: area of quad. BCED $= 1:8$ Ans

2) In $\triangle ABC$, $\angle ABC = \angle DAC$, $AB = 8cm$, $AC = 4cm$, $AD = 5cm$.

i) P.T. $\triangle ACD \sim \triangle BCA$

ii) find BC & CD

iii) find $ar(\triangle ACD) : ar(\triangle ABC)$.



Pf: i) In $\triangle ACD$ & $\triangle BCA$

$\angle DAC = \angle ABC$ (Given)

$\angle ACD = \angle BCA$ (Common)

$\Rightarrow \triangle ACD \sim \triangle BCA$ (by AA Axm)

$$\frac{AC}{BC} = \frac{CD}{CA} = \frac{AD}{BA}$$

$$\Rightarrow \underbrace{\frac{4}{BC} = \frac{CD}{4} = \frac{5}{8}}$$

$$\frac{4}{BC} = \frac{5}{8}$$

$$\frac{32}{5} = BC \Rightarrow BC = 6.4cm$$

$$CD = \frac{5}{\cancel{8}_2} \times \cancel{4}$$

$$CD = 2.5cm$$

ii) $area(\triangle ACD) : area(\triangle BCA) = \left(\frac{AD}{AB}\right)^2 = \left(\frac{5}{8}\right)^2$

$$= \frac{25}{64} \text{ Ans}$$

3) In $\triangle ABC$, $AP : PB = 2 : 3$.

PO is parllel to BC & is extended to Q so that CQ is parllel to BA. Find

i) area $\triangle APO$: area $\triangle ABC$

ii) area $\triangle APO$: area $\triangle CQO$