

Lecture-ISimilarity (As a size transformation)

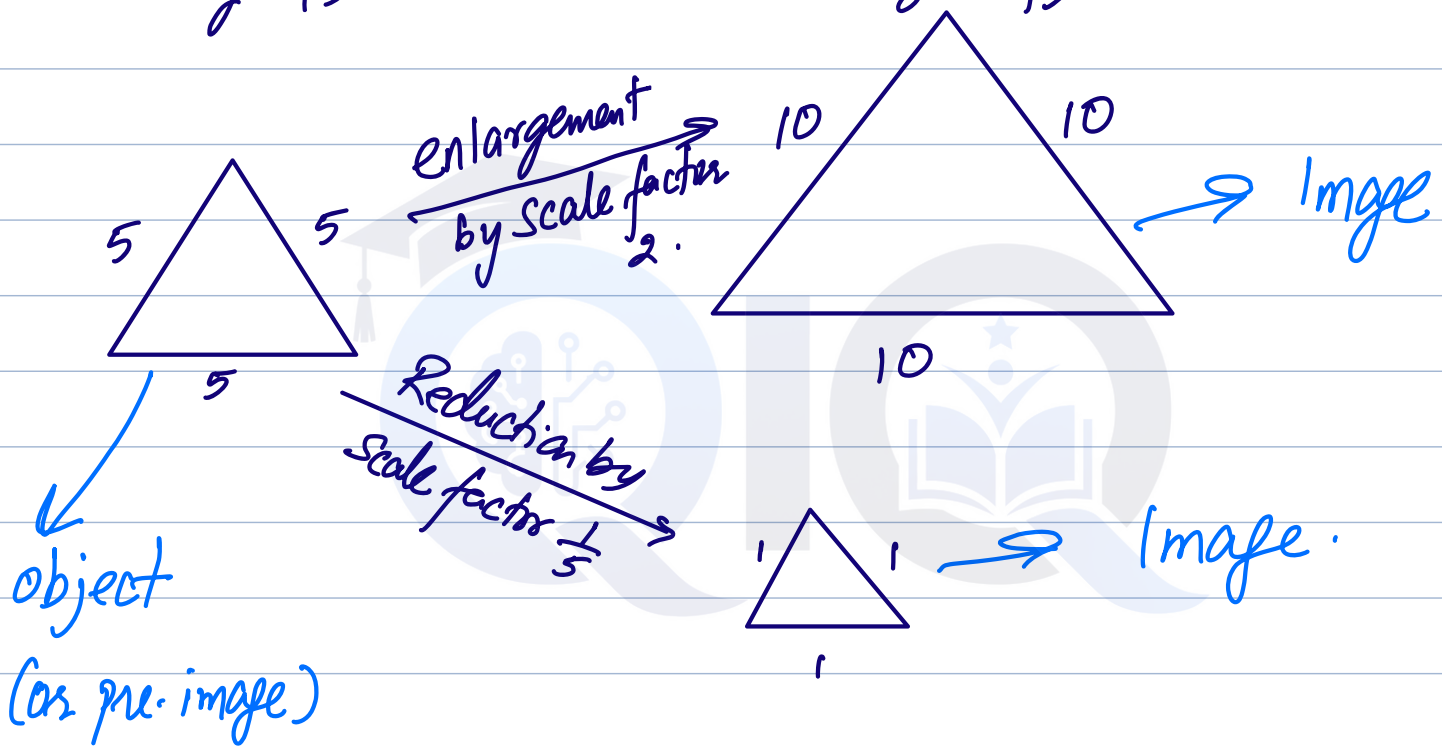
#

i) Enlargement

ii) Reduction

Size Transformation

It is the process in which a given figure is enlarged or reduce by a scale factor k , such that the resulting figure is similar to the given figure.



Properties of Size Transformation

- ✓ (i) In a size transformation, the shape of the given figure is preserved.
Thus, angle, perpendicularity, parallelism, etc., are preserved.
- ✓ (ii) Let k be the scale factor of a given size transformation. Then,
 $k > 1 \Rightarrow$ The transformation is an **enlargement**.
 $k < 1 \Rightarrow$ The transformation is a **reduction**.
 $k = 1 \Rightarrow$ The transformation is an **identity transformation**.
- ✓ (iii) Each side of the resulting figure = k times the corresponding side of the given figure.
- ✓ (iv) Area of the resulting figure = $k^2 \times$ (Area of the given figure).
- (v) In case of solids, we have
Volume of the resulting figure = $k^3 \times$ (Volume of the given figure).

Model

The model of a plane figure and the actual figure are similar to one another.
Let the model of a plane figure be drawn to the scale $1 : p$.

Then, Scale Factor, $k = \frac{1}{p}$.

- (i) Length of the model = $k \times$ (Length of the actual figure).
- (ii) Area of the model = $k^2 \times$ (Area of the actual figure).
- (iii) Volume of the model = $k^3 \times$ (Volume of the actual figure).

Map

Let the map of a plane figure be drawn to the scale $1 : p$.

Then, Scale Factor, $k = \frac{1}{p}$.

- (i) Length in the map = $k \times$ (Actual length).
- (ii) Area in the map = $k^2 \times$ (Actual area).

On a map drawn to a scale of 1 : 50000, a rectangular plot of land ABCD has the following dimensions : AB = 6 cm, BC = 8 cm and all angles are right angles. Find:

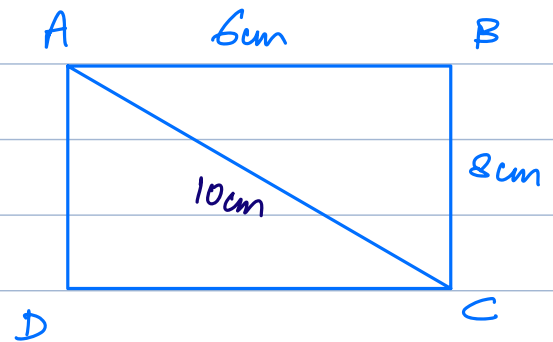
- (i) the actual length of the diagonal distance AC of the plot in km
- (ii) the actual area of the plot in sq km

(2018)

Sol: $K = \frac{1}{50,000}$

$\therefore$ Length of the map = K (Actual length)

$\therefore 10 = \frac{1}{50,000}$ (Actual length of AC)



$\Rightarrow$ Actual length of AC = 500,000 cm = 5 km

$\left. \begin{array}{l} 1 \text{ km} = 1000 \text{ m} \\ 1 \text{ m} = 100 \text{ cm} \end{array} \right\} \Rightarrow 1 \text{ km} = 100,000 \text{ cm}$

$\therefore$ Area of the map = K^2 (Actual Area)

$\therefore 48 \text{ cm}^2 = \left(\frac{1}{50,000} \right)^2$ (Actual Area)

Actual Area = $48 \times 50,000 \times 50,000 \text{ cm}^2$
 $= \frac{48 \times \cancel{50,000} \times \cancel{50,000}}{\cancel{100,000} \times \cancel{100,000}} \text{ Km}^2 = 12 \text{ Km}^2$

A model of a ship is made to a scale 1 : 300.

- (i) The length of the model of the ship is 2 m. Calculate the length of the ship.
- (ii) The area of the deck of the ship is 180000 m^2 . Calculate the area of the deck of the model.
- (iii) The volume of the model is 6.5 m^3 . Calculate the volume of the ship. (2016)

Soln: $K = \frac{1}{300}$

i)

$$\therefore \text{length of the model} = K (\text{Actual length})$$

$$\therefore 2\text{m} = \frac{1}{300} (\text{length of the ship}) \Rightarrow \text{length of the ship} = 600\text{m}.$$

ii) Area of the deck of the model = $\left(\frac{1}{300}\right)^2 (180000 \text{ m}^2) = \frac{180000}{300 \times 300} \text{ m}^2$
 $= 2 \text{ m}^2$.

iii) $6.5 \text{ m}^3 = \left(\frac{1}{300}\right)^3 \text{ Volume of the ship}$

$$6.5 \times 300 \times 300 \times 300 \text{ m}^3 = \text{Volume of the ship}.$$

$$\Rightarrow \boxed{175500000 \text{ m}^3 = \text{Volume of the ship}} \quad \underline{\text{Ans}}$$

(OR) Volume of the ship = $\frac{175500000}{1000 \times 1000 \times 1000} \text{ km}^3$

$$= \underline{\underline{0.1755 \text{ km}^3}} \quad \underline{\text{Ans}}$$

The model of a building is constructed with scale factor 1 : 30.

- (i) If the height of the model is 80 cm, find the actual height of the building in metres.
- (ii) If the actual volume of the tank on the top of the building is 27m^3 , find the volume of the tank on the top of the model. (2009, 2019)

Sol:- i) $K = \frac{1}{30} \rightarrow 80\text{ cm} = \frac{1}{30} (\text{Actual height})$

$$\text{Actual height} = 2400\text{ cm} = \underline{24\text{ m}} \quad \underline{\text{Ans}}$$

ii) Volume of the model = $\left(\frac{1}{30}\right)^3 (27) \text{ m}^3 = \frac{27 \times 100 \times 100 \times 100 \text{ cm}^3}{30 \times 30 \times 30}$

$$= \underline{1000 \text{ cm}^3} \quad \underline{\text{Ans}}$$

$$1\text{ cm} = \frac{1}{100} \text{ m} \rightarrow 1\text{ cm}^3 = \left(\frac{1}{100}\right)^3 \text{ m}^3 \Rightarrow 1000\text{ cm}^3 = \frac{1000}{100 \times 100 \times 100} \text{ m}^3$$
$$= \frac{1}{1000} \text{ m}^3 = \underline{0.001 \text{ m}^3} \quad \underline{\text{Ans}}$$

✓ 5. On a map, drawn to a scale of 1 : 25000, a triangular plot LMN of land has the following measurements :

LM = 6 cm, MN = 8 cm and $\angle LMN = 90^\circ$. Calculate :

(i) the actual lengths of MN and LN in kilometres,

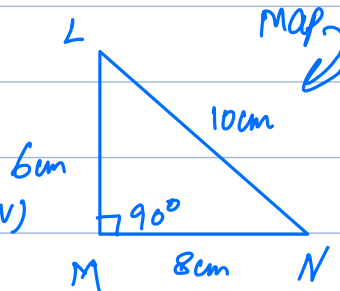
(ii) the actual area of the plot in sq. km.

Sol'n

$$K = \frac{1}{25000}$$

i) Length of map MN = K (Actual length of MN)

$$8 = \frac{1}{25000} (\text{Actual length of MN})$$



$$\text{Actual length of MN} = (25000 \times 8) \text{ cm} = 200,000 \text{ cm} = 2 \text{ km}$$

$$\text{Length of map LN} = K (\text{Actual length of LN})$$

$$\text{Actual length of LN} = 10 \times 25000 = 250,000 \text{ cm} = 2.5 \text{ km}$$

ii) Area of map = K^2 (Actual Area)

$$\begin{aligned} \Rightarrow \text{Actual} &= 25000 \times 25000 \times 24 \text{ cm}^2 = \frac{25000 \times 25000 \times 24}{100,000 \times 100,000} \text{ km}^2 \\ &= \underline{\underline{1.5 \text{ km}^2}} \end{aligned}$$