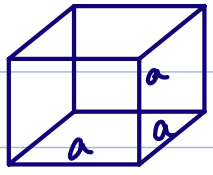
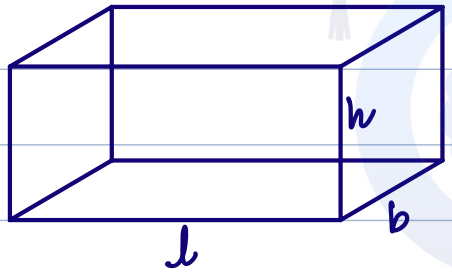
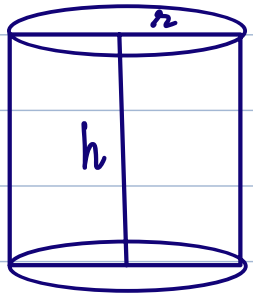
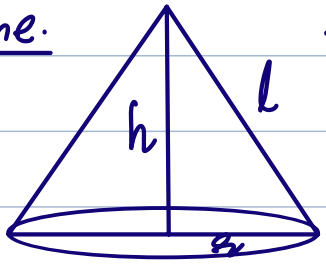
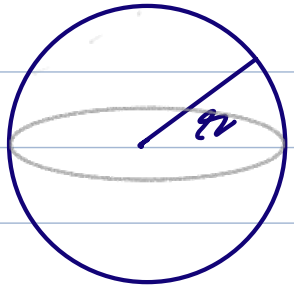
Volume & Surface Area.

Lecture-I

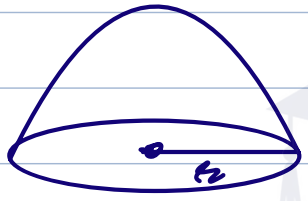
Volume = Area of Cross-Section × height

Shape	Volume	Curved Surface area (Lateral Surface area)	Total Surface area
1) Cube 	a^3	$4a^2$	$6a^2$.
2) Cuboid 	lbh	$2(l+b)h$	$2(lb+bh+lh)$
3) Cylinder. 	$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$.
4) Cone. 	$\frac{1}{3}\pi r^2 h$	πrl $(l = \sqrt{h^2+r^2})$ ↓ Slant height	$\pi rl + \pi r^2$ or $\pi r(l+r)$.

5) Sphere. $\frac{4}{3}\pi r^3$ $4\pi r^2$ $4\pi r^2$.



6) Hemi-sphere. $\frac{2}{3}\pi r^3$ $2\pi r^2$ $3\pi r^2$



7) Hollow Cylinder $\pi R^2 h - \pi r^2 h$ $2\pi Rh + 2\pi rh$ $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

(or) $\pi h(R^2 - r^2)$

$\pi h(R-r)(R+r)$

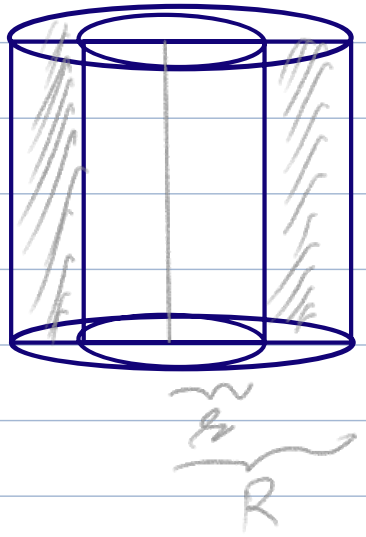


Illustration.

1) The circumference of the base of a cylindrical vessel is 132cm. & its height is 25cm. Find the radius & volume of the cylinder.

Soln:- Circumference = 132cm $\Rightarrow 2\pi r = 132$

$\Rightarrow r = \frac{132 \times 7}{2 \times 22} = 3 \times 7 = 21$cm

$r = 21$cm

Volume $= \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 25 = 66 \times 525$

$= 34650\ cm^3.$

2) Find the number of coins 1.5cm in diameter and 0.2 cm thick to be melted to form a right circular cylinder of height 10cm & diameter 4.5cm.

$\hookrightarrow \frac{45}{10} = \frac{9}{2}.$

Soln:- Let the no. of coins be n.

$$n = \frac{\text{Volume of circular cylinder}}{\text{Volume of one coin.}}$$

Coin:- $r = \frac{3}{4}$ cm, $h = \frac{1}{5}$ cm

Cylinder :- $R = \frac{9}{4}$, $H = 10$

$$n = \frac{\pi \times \frac{9}{4} \times \frac{9}{4} \times 10}{\pi \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{5}} = \frac{90}{1/5}$$

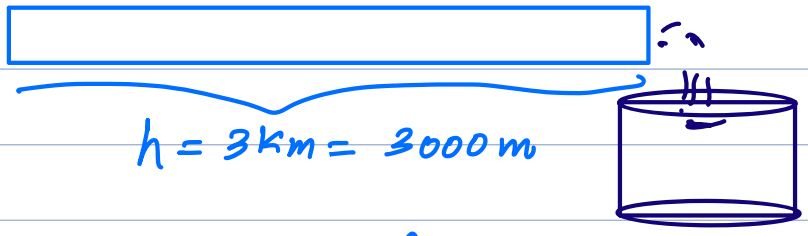
$n = 450$ Ans

3) Water is flowing at the rate of 3km/hr through a circular pipe of 20cm internal diameter into a circular cistern of diameter 10m & depth 2m. In how much time will the cistern be filled?

$d = 20cm = 0.2\,m$

$R = 0.1m = \frac{1}{10}m$.

Sol^n:.



$h = 3km = 3000\,m$

$\rightarrow R = 5m$

$H = 2m$.

Time frame of 1hr.

Volume of water flown out in 1hr. $= \pi\left(\frac{1}{10}\right)^2(3000)$

$= \pi \times \frac{1}{10} \times \frac{1}{10} \times 3000 = 30\pi\ m^3$.

Volume of Cistern $= \pi R^2 H$

$= \pi(5)^2(2) = 50\pi\ m^3$.

Time taken $= \left(\frac{50\pi\ m^3}{30\pi\ m^3}\right) hrs = \frac{5}{3}$ hrs.

$= 1\frac{2}{3}$ hrs $=$ 1hr 40min

Lecture-II

4) Water is flowing at the rate of 8m per second through a circular pipe whose internal diameter is 2cm, into a cylindrical tank, the radius of whose base is 40 cm. Determine the increase in the water level in 30 minutes.

Sol^n:-

Volume of water flown out of the pipe in 1sec
$= \pi(1)^2(800) = 800\pi \text{ cm}^3$.

Volume of water flown out of the pipe in 1800 sec (30 mins) $= (1800 \times 800)\pi \text{ cm}^3$.

Let the height of cylindrical be H. & R = 40cm

$\Rightarrow \pi(40)^2 H = (1800 \times 800)\pi \text{ cm}^3$.

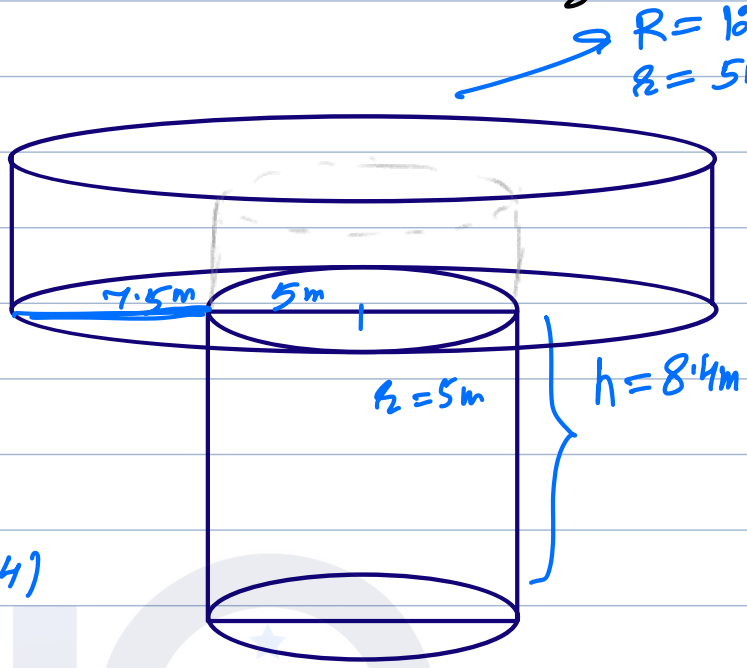
$H = \frac{1800 \times 800}{40 \times 40}$

$H = 900 \text{ cm}$

or

$H = 9\text{m}$

5) A well with 10m inside diameter is dug 8.4m deep. Earth taken out of it is spread all around it to a (Soil) width of 7.5m to form an embankment. Find the height of embankment.



Sol^{n}:. Let the height of embankment be h

Volume of embankment = volume of soil dugged out. (Volume of cylinder)

$$\pi h\left[(12.5)^2-(5)^2\right] = \pi(5)^2(8.4)$$

$$h = \frac{(25)\times(8.4)}{(12.5+5)(12.5-5)} = \frac{(25)\times(8.4)}{(17.5)(7.5)}.$$

$$\boxed{h = 1.6\,m}$$ Ans.

6) The volume of a Conical tent is $1232\ m^3$ & the area of the base floor is $154\ m^2$. Find i) the radius of the floor.

ii) the height of the tent

iii) the length of Canvas to cover this Conical tent, it is being given that the width of the Canvas is 2m.

Sol:

$\pi r^2 = 154$

$r^2 = \frac{154}{\frac{22}{7}} \cdot 7 = \frac{7}{1/7} = 49$

$\boxed{r = 7m}$ Ans

Volume $= 1232\ m^3$

$\Rightarrow \frac{1}{3}\pi(7)^2 h = 1232 \Rightarrow h = \frac{1232 \times 3}{\frac{22}{7} \times 7 \times 7}$

$\Rightarrow \boxed{h = 24m}$ Ans

$l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = 25$

Area of Canvas = Curved surface area of Cone ($\pi r l$)

$(Length)(2) = \pi(7)(25)$

$Length = \frac{22}{7} \times 7 \times \frac{25}{2} = 275\ m$ Ans

7) A girl fills a cylindrical bucket 32 cm in height & 18cm in radius with sand. She empties the bucket on the ground & makes a conical heap of the sand. If the height of the conical heap is 24cm. find i) Radius
ii) Slant height.

Soln:- Volume of Cone = Volume of sand (volume of cylindrical bucket)

$$\frac{1}{3}\pi r^2(24) = \pi(18)^2(32)$$

$$r^2 = \frac{18\times18\times32}{8} \Rightarrow r^2 = (18\times2)\times(18\times2) = 36\times36$$

$$\Rightarrow \boxed{r = 36\text{ cm}}$$

$$l = \sqrt{(24)^2+(36)^2} = \sqrt{576+1296} = \sqrt{1872} = 12\sqrt{13}\text{ cm}$$

8) A Conical tent is to accomodate 77 persons. Each person must have $16 m^3$ air to breathe. Given the radius of the tent as 7m, find the height of the tent & also its Curved surface area.

Soln:-

Volume of Cone = $(77 \times 16)\ m^3$.

$$\frac{1}{3}\pi r^2 h = 77 \times 16 \Rightarrow \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times h = 77 \times 16$$

$$\Rightarrow \boxed{h = 24m}$$

$$CSA = \pi r l = \frac{22}{7} \times 7 \times \sqrt{(24)^2 + (7)^2} = 22 \times 25 = 550 m^2.$$

9) The height of a Cone is 40cm. A small Cone is cut off at the top by a plane parallel to its base. If its volume be $\frac{1}{64}$ to the volume of the given Cone, at what height above the base is the section cut?

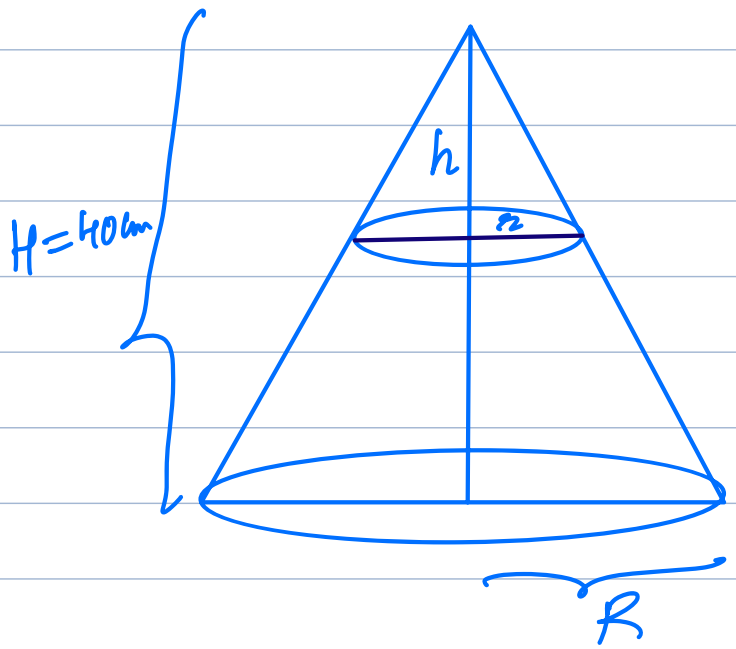
To find $(H-h)$

Soln:-

$$\left\{\frac{h}{H} = \frac{r}{R}\right\}$$

$$\frac{h}{40} = \frac{r}{R}$$

$$\boxed{h = \frac{40r}{R}}$$



Volume of Smaller Cone $= \frac{1}{64}$ Volume of larger Cone.

$$\Rightarrow \frac{1}{3}\pi(r^2)(h) = \frac{1}{64}\frac{\pi(R)^2(H)}{3}$$

$$\Rightarrow r^2 \times \frac{40r}{R} = \frac{1}{64}(R^2)(40) \Rightarrow r^3 = \frac{R^3}{64}$$

$$\Rightarrow R^3 = 64r^3 \Rightarrow R = 4r \Rightarrow h = \frac{40r}{4r} \Rightarrow 10$$

$h = 10\,cm$

$$H - h = (40 - 10)\,cm = 30\,cm.$$