Section A

1.

(b) $\begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

Explanation:

$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$\Rightarrow$ A = (A')' $= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$\therefore$ A + 2B $= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$\Rightarrow$ (A + 2B)' $= \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

2.

(d) AB = BA= I

Explanation:

A = B^{-1}

B = A^{-1}

We know that

AA^{-1} = I

(Given B = A^{-1})

AB = I ...(i)

We know that

BB^{-1} = I

(Given A = B^{-1})

BA = I ...(ii)

From (i) and (ii)

AB = BA = I

3. **(a)** 47

Explanation:

47

$\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

= 2(1-8)-7(1-10)+1(8-10)

= 2(-7) - 7(-9) +1(-2)

= -14 +63-2

= -16 +63

= 47

4.

(b) 2^x (log 2)

Explanation:

Given that $y = 2^x$

Taking log both sides, we get

$\log_e y = x \log_e 2$ (Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e 2$ or $\frac{dy}{dx} = \log_e 2 \times y$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

5. **(a)** 10

Explanation:

Determinant of these point should be zero

$$\begin{vmatrix} -1 & 3 & 2 \\ -4 & 2 & -2 \\ 5 & 5 & \lambda \end{vmatrix} = 0$$

$-1(2\lambda + 10) - 3(-4\lambda + 10) + 2(-20 - 10) = 0$

$10\lambda = 10 + 30 + 60 = 100$

$\lambda = 10$

6.

(b) 1

Explanation:

$x^2 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^3$

$x^3 \frac{d^2y}{dx^2} = x^3\left(\frac{dy}{dx}\right)^3 - y^3 - 3x^2y\left(\frac{dy}{dx}\right)^2 + 3y^2x\frac{dy}{dx}$

$x^3 \frac{d^2y}{dx^2} - x^3\left(\frac{dy}{dx}\right)^3 + 3x^2y\left(\frac{dy}{dx}\right)^2 + 3xy^2\frac{dy}{dx} + y^3 = 0$

Order $= 2$

degree $= 1$

7. **(a)** R

Explanation:

Corner points	Value of Z = 2x + 5y
P(0, 5)	Z = 2(0) + 5(5) = 25
Q(1, 5)	Z = 2(1) + 5(5) = 27
R(4, 2)	Z = 2(4) + 5(2) = 18 $\rightarrow$ Minimum
S(12, 0)	Z = 2(12) + 5(0) = 24

Thus, minimum value of Z occurs ar R(4, 2)

8.

(b) $\frac{2\pi}{9}$

Explanation:

$\cos^{-1}(\cos(680°))$

$= \cos^{-1}[\cos(720° - 40°)]$

$= \cos^{-1}[\cos(-40°)]$

$= \cos^{-1}[\cos(40°)]$

$= 40°$

$= \frac{2\pi}{9}$.

9.

(c) 0

Explanation:

If f is an odd function,
$\int_{-a}^{a} f(x)dx = 0$
as, $\int_0^a f(x)dx = -\int_{-a}^{0} f(x)dx$
f(x) = $\sin^5$ x
f(-x) = $\sin^5$ (-x)
Therefore, f(x) is odd number
$\int_{-\pi}^{\pi} \sin^5 x dx = 0$

10.
(b) x = 2, y = 3
Explanation:
We have, $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$
$\Rightarrow$ 4x = x + 6 $\Rightarrow$ x = 2
and 4x = 7y - 13
$\Rightarrow$ 8 = 7y - 13
$\Rightarrow$ y = 3

11. **(a)** the problem is to be re-evaluated
Explanation:
The optimisation of the objective function of a LPP is governed by the constraints. Therefore, if the constraints in a linear programming problem are changed, then the problem needs to be re-evaluated.

12.
(c) -1
Explanation:
We know that,
$|\vec{a}+\vec{b}+\vec{c}|^2 = |\overrightarrow{a}^2| + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$ ... (i)
Since,
$\vec{a}$ is perpendicular to $\vec{b}$
$\Rightarrow \vec{a}\cdot\vec{b} = 0$
And according to question
$|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$
We can rewrite equation (i) as
$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 2\cos\beta + 2\cos\alpha$
1 = 1 + 1 + 1 + 0 + 2 (cos α + cos β)
1=3+2$(cos\alpha + cos\beta)$
-2=2$(cos\alpha + cos\beta)$
$\Rightarrow$cos α + cosβ = -1

13. **(a)** 11
Explanation:
11

14.
(b) $\frac{1}{35}$
Explanation:
Consonants can be placed in 4! ways.
Vowels can be placed in between consonants in 4! ways
$\therefore$ Total no. of ways in which the consonants and vowels occur alternatively = $2 \times 4! \times 4!$
Also, total no. of ways in which the letters of the word 'ORIENTAL' can be arranged = 8!
$\therefore$ Required probability = $\frac{2\times4!\times4!}{8!} = \frac{1}{35}$

15. **(a)** $y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

Explanation:

$y^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

16.

(b) $\pm\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Explanation:

Since, unit vector perpendicular to both $\vec{a}$ and $\vec{b}$ = $\pm\frac{a\times b}{|a\times b|}$

$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}$

$= \hat{i}[1+1] - \hat{j}[-1-1] + q\hat{k}[1-1]$

$= 2\hat{i} + 2\hat{j} + 0\hat{k} = 2(\hat{i}+\hat{j})$

and $|\vec{a}\times\vec{b}| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore$ Required unit vector

$= \pm\frac{2(\hat{\mathbf{i}}+\hat{\mathbf{j}})}{2\sqrt{2}} = \pm\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$

17.

(b) continuous $\forall$ x $\in$ R and not differentiable at x = 0

Explanation:

f(x) = x - |x| = g(x) + h(x)

where y(x) = x, h(x) = -|x|

As g(x) and h(x) are both continuous $\forall$ x $\in$ R

$\therefore$ f(x) is continuous $\forall$ x $\in$ R

And g(x) is differentiable $\forall$ x $\in$ R

but h(x) is not differentiable at x = 0

$\therefore$ f(x) = g(x) + h(x) is not differentiable at x = 0

18.

(b) $\frac{4}{3}$

Explanation:

$\frac{4}{3}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Let f(x) = x^2 - 8x + 17

$\therefore$ f'(x) = 2x - 8

So, f'(x) = 0, gives x = 4

Here x = 4 is the critical number

Now, f''(x) = 2 > 0 , $\forall$ x

So, x = 4 is the point of local minima.

$\therefore$ Minimum value of f(x) at x = 4,

f(4) = 4 $\times$ 4 - 8 $\times$ 4 + 17 = 1

Hence, we can say that both Assertion and Reason are true and Reason is the correct explanation of the Assertion.

20.

(c) A is true but R is false.

Explanation:

Assertion: Here, f : R $\rightarrow$ R is given by

f(x) = |x| = $\begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

It is seen that f(-1) = |-1| = 1, f(1) = |1| = 1

Therefore, f(-1) = f(1) but -1 $\neq$ 1

Therefore, f is not one-one.

Now, consider -1 $\in$ R

It is known that f(x) = |x| is always non-negative

Thus, there does not exist any element x in domain R such that f (x) = |x| = -1.

Therefore, f is not onto.

Hence, the modulus function is neither one-one nor onto.

Reason: f : R $\rightarrow$ R, f(x) = $\begin{cases} 1, & \text{if} \quad x > 0 \\ 0, & \text{if} \quad x = 0 \\ -1, & \text{if} \quad x < 0 \end{cases}$

It is seen that f(1) = f(2) = 1 but 1 $\neq$ 2.

Therefore, f is not one-one

Now, as f(x) takes only three values (1, 0 or -1), therefore for the element -2 in codomain R, there does not exist any x in domain R such that f(x) = -2

Therefore, f is not onto. Hence, the Signum function is neither one-one nor onto.

Section B

21. Let $\sin^{-1}\left(\frac{-1}{2}\right) = y$

$\Rightarrow \sin y = -\frac{1}{2}$

$\Rightarrow \sin y = -\sin\frac{\pi}{6}$

$\Rightarrow \sin y = \sin\left(-\frac{\pi}{6}\right)$

Since, the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is $-\frac{\pi}{6}$.

OR

Let $\cos^{-1}\left(\frac{1}{2}\right) = x$. Then, $\cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$.

$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$. Then, $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + \frac{2\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

22. The function is f (x) = sin x

Then, $f'(x) = cosx$

Since for each x ϵ $\left(\frac{\pi}{2}, \pi\right)$, cos x < 0, we have $f'(x) < 0$

Therefore, f is strictly decreasing in$\left(\frac{\pi}{2}, \pi\right)$.

23. Here,

$y = \frac{2}{3}x^3 + 1$

Differentiating both sides with respect to t,

$\Rightarrow \frac{dy}{dt} = 2x^2\frac{dx}{dt}$

$\Rightarrow 2\frac{dx}{dt} = 2x^2\frac{dx}{dt}\left[\because \frac{dy}{dt} = 2\frac{dx}{dt}\right]$

$\Rightarrow x = \pm 1$

Substituting the value of x = 1 and x= -1 in y = $\frac{2}{3}$$x^3$ + 1, we get

$\Rightarrow y = \frac{5}{3}$ and $y = \frac{1}{3}$

So , the points are $\left(1, \frac{5}{3}\right)$ *and* $\left(-1, \frac{1}{3}\right)$

OR

Given $R(x) = 13x^2 + 26x + 15$

Now, $MR = \frac{d}{dx}(R(x)) = 26x + 26$

$MR]_{x=17} = 25 \times 17 + 26$

= 425 + 26

= 451

24. We have

$I = \int e^x \cdot \left\{\frac{2+\sin 2x}{1+\cos 2x}\right\} dx$

$= \int e^x \cdot \left\{\frac{2}{(1+\cos 2x)} + \frac{\sin 2x}{(1+\cos 2x)}\right\} dx$

$= \int e^x \cdot \left\{\frac{2}{2\cos^2 x} + \frac{2\sin x \cos x}{2\cos^2 x}\right\} dx$ $=\int e^x.|\sec^2 x + \tan x|\ dx$

$= \int e^x.(\tan x + \sec^2 x)\ dx$

$= \int e^x .\{f(x) + f'(x)\}\ dx$ where $f(x) = \tan x$ a $f'(x) = \sec^2 x$

$= e^x f(x) + C = e^x \tan x + C$

25. Given: $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

Expanding along first row, $1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$

= {-9(-12)} - {-18 - (-15)} - 2(8 - 5)

= -9 + 12 - (-18 + 15) -2(3)

= 3(-3) - 6

= 3 + 3 - 6 = 0

Section C

26. let the given integral be

$l = \int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$

Let $3 \sin x + 2 \cos x = \lambda \frac{d}{dx}(3 \cos x + 2 \sin x) + \mu (3 \cos x + 2 \sin x)$

i.e. $3 \sin x + 2 \cos x = \lambda (- 3 \sin x + 2 \cos x) + \mu(3 \cos x + 2 \sin x)$

Comparing the coefficients of sin x and cos x on both sides, we get

$- 3\lambda + 2\mu = 3$ and $2\lambda + 3\mu = 2 \Rightarrow \mu = \frac{12}{13}$ and $\lambda = -\frac{5}{13}$

$\therefore \quad I = \int \frac{\lambda(-3\sin x + 2\cos x) + \mu(3\cos x + 2\sin x)}{3\cos x + 2\sin x} dx$

$\Rightarrow \quad I = \mu \int 1 \cdot dx + \lambda \int \frac{-3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$

$\Rightarrow \quad I = \mu x + \lambda \int \frac{dt}{t}$, where $t = 3 \cos x + 2 \sin x$

$\Rightarrow \quad I = \mu x + \lambda \log |t| + C$

$= \frac{12}{13}x - \frac{5}{13} \log |3 \cos x + 2 \sin x| + C$

27. A white ball and a red ball can be drawn in three mutually exclusive ways:

i. Selecting bag I and then drawing a white and a red ball from it
ii. Selecting bag II and then drawing a white and a red ball from it
iii. Selecting bag III and then drawing a white and a red ball from it

Consider the following events:

E_1 = Selecting bag I

E_2 = Selecting bag II

E_3 = Selecting bag II

A = Drawing a white and a red ball

It is given that one of the bags is selected randomly.

$\therefore P(E_1) = \frac{1}{3}$

$P(E_2) = \frac{1}{3}$

$P(E_3) = \frac{1}{3}$

Now,

$P\left(\frac{A}{E_1}\right)=\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}=\frac{3}{15}$
$P\left(\frac{A}{E_2}\right)=\frac{{}^2C_1\times{}^1C_1}{{}^4C_2}=\frac{2}{6}$
$P\left(\frac{A}{E_3}\right)=\frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2}=\frac{12}{66}$
Using the law of total probability, we get
Required probability = P(A) = $P(E_1)P(\frac{A}{E_1})$ + $P(E_2)P(\frac{A}{E_2})$ + $P(E_3)P(\frac{A}{E_3})$
$=\frac{1}{3}\times\frac{3}{15}+\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{12}{66}$
$=\frac{1}{15}+\frac{1}{9}+\frac{2}{33}$
$=\frac{33+55+30}{495}=\frac{118}{495}$

28. Given $I=\int\frac{2x}{(1+x^2)(x^2+3)}dx$

Put x^2 = t ⇒ 2xdx = dt
$\therefore I=\int\frac{dt}{(t+1)(3+t)}$
Using partial fractions,
$\frac{1}{(t+1)(3+t)}=\frac{A}{(1+t)}+\frac{B}{(3+t)}$ ...(i)
⇒ 1 = A(3 + t) + B(1 + t)
Putting t = -3, we get
$\Rightarrow 1=-2B\Rightarrow B=-\frac{1}{2}$
Putting t = -1, we get
⇒1 = 2A ⇒ $A=\frac{1}{2}$
On putting A = $\frac{1}{2}$ and B = $-\frac{1}{2}$ in Eq. (i), we get
$\frac{1}{(1+t)(3+t)}=\frac{\frac{1}{2}}{1+t}+\frac{\frac{(-1)}{2}}{3+t}$
Integrating both sides w.r.t. to t ,
$\Rightarrow\int\frac{1}{(1+t)(3+t)}dt=\frac{1}{2}\int\frac{1}{1+t}dt-\frac{1}{2}\int\frac{1}{3+t}dt$
$=\frac{1}{2}\log|1+t|-\frac{1}{2}\log|3+t|+C$
$=\frac{1}{2}\log\left|1+x^2\right|-\frac{1}{2}\log\left|3+x^2\right|+C[\text{ put }t=x^2]$
$\therefore\quad I=\frac{1}{2}\log\left|\frac{1+x^2}{3+x^2}\right|+C$ [∵ log m - log n = log$\frac{m}{n}$]

OR

We have,
$\frac{1}{1+\cot x}=\frac{1}{1+\frac{\cos x}{\sin x}}=\frac{\sin x}{\sin x+\cos x}$
$\therefore\int_0^{\frac{\pi}{2}}\frac{1}{1+\cot x}dx=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx$
Let
$I=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx$ ..(i)
so, using property of defnite integrals we have
$I=\int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}dx\ \left[\because\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$
$=\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$ ..(ii)
Adding (i) & (ii), we get
$2I=\int_0^{\frac{\pi}{2}}\frac{\sin x}{\sin x+\cos x}dx+\int_0^{\frac{\pi}{2}}\frac{\cos x}{\sin x+\cos x}dx$
$=\int_0^{\frac{\pi}{2}}\frac{\sin x+\cos x}{\sin x+\cos x}dx$
$=\int_0^{\frac{\pi}{2}}dx$
$=[x]_0^{\frac{\pi}{2}}$
$=\left[\frac{\pi}{2}-0\right]$
$\Rightarrow\ I=\frac{\pi}{4}$

29. $\left(x^2+y^2\right) dy = xydx$

$\Rightarrow \int \frac{x}{y} dy + \int \frac{y}{x} dy = \int dx$

$\Rightarrow x \log y + \frac{y^2}{2x} = x + c$

Now, at x = 1; y = e

$x \log y + \frac{y^2}{2x} = x + c \Rightarrow x + \frac{e^2}{2} = x + c \Rightarrow c = \frac{e^2}{2}$

Now at x = x_0; y = e

$x_0 \log e + \frac{e^2}{2x_0} = x_0 + \frac{e^2}{2} \Rightarrow \frac{e^2}{2x_0} = \frac{e^2}{2} \Rightarrow x_0 = 1$

OR

The given differential equation may be written as

$\frac{dy}{dx} = \frac{3xy}{(y^2-x^2)}$ ...(i)

On dividing the Nr and Dr of RHS of (i) by x^2, we have,

$\frac{dy}{dx} = \frac{3\left(\frac{y}{x}\right)}{\left\{\left(\frac{y}{x}\right)^2-1\right\}} = f\left(\frac{y}{x}\right)$

Therefore, the given differential equation is homogeneous.

Putting y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ in (i), we have,

$v + x\frac{dv}{dx} = \frac{3v}{(v^2-1)}$

$\Rightarrow \quad x\frac{dv}{dx} = \left\{\frac{3v}{(v^2-1)} - v\right\} = \frac{(4v-v^3)}{(v^2-1)}$

$\Rightarrow \quad \frac{(v^2-1)}{(4v-v^3)} dv = \frac{1}{x} dx$

$\Rightarrow \int \frac{(v^2-1)}{v(2-v)(2+v)} dv = \int \frac{1}{x} dx$ ...(ii)

Let $\frac{(v^2-1)}{v(2-v)(2+v)} = \frac{A}{v} + \frac{B}{(2-v)} + \frac{C}{(2+v)}$

Therefore, $(v^2 - 1) \equiv A(2 - v)(2 + v) + Bv(2 + v) + Cv(2 - v)$ .. (ii)

Putting v = 0 on each side of (iii), we get A = $\frac{-1}{4}$

Putting u = 2 on each side of (iii), we get B = $\frac{3}{8}$

Putting v = -2 on each side of (iii), we get C = $\frac{-3}{8}$

$\therefore \frac{(v^2-1)}{v(2-v)(2+v)} = \frac{-1}{4v} + \frac{3}{8(2-v)} - \frac{3}{8(2+v)}$ ...(iv)

Putting these values from (iv) in (ii), we have,

$-\frac{1}{4}\int \frac{dv}{v} + \frac{3}{8}\int \frac{dv}{(2-v)} - \frac{3}{8}\int \frac{dv}{(2+v)} = \int \frac{1}{x} dx$

$\Rightarrow \int \frac{1}{x} dx + \frac{1}{4}\int \frac{dv}{v} + \frac{3}{8}\int \frac{-dv}{(2-v)} + \frac{3}{8}\int \frac{dv}{(2+v)}$ = log $|C_1|$ where C_1 is an arbitrary constant

$\Rightarrow$ log |x| + $\frac{1}{4}$ log |v| + $\frac{3}{8}$ log |2 - v| + $\frac{3}{8}$ log |2 + v| = log $|C_1|$

$\Rightarrow$ 8 log |x| + 2 log |v| + 3 log |2 - v| + 3 log |2 + v| = 8 log $|C_1|$

$\Rightarrow$ log $|x^8 v^2 (2 - v)^3 (2 + v)^3|$ = log (C_1^8)

$\Rightarrow$ $|x^8 v^2 (2 - v)^3 (2 + v)^3| = C_1^8$ = C (say)

$\Rightarrow \quad x^6 y^2 \left(2 - \frac{y}{x}\right)^3 \left(2 + \frac{y}{x}\right)^3$ = C

This is the required solution of given differential equation.

30. Given,

$\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0} \Rightarrow \vec{a} + \vec{b} = -\vec{c}$

$\Rightarrow (\vec{a} + \vec{b}) \times \vec{b} = (-\vec{c}) \times \vec{b}$

$\Rightarrow (\vec{a} \times \vec{b}) + (\vec{b} \times \vec{b}) = (-\vec{c}) \times \vec{b}$ [by the distributive law]

$\Rightarrow (\vec{a} \times \vec{b}) + \overrightarrow{0} = (\vec{b} \times \vec{c})$ $[\because \quad \vec{b} \times \vec{b} = \overrightarrow{0}$ and $(-\vec{c}) \times \vec{b} = \vec{b} \times \vec{c}]$

$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ...(i)

Also, $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0} \Rightarrow \vec{b} + \vec{c} = -\vec{a}$

$\Rightarrow (\vec{b} + \vec{c}) \times \vec{c} = (-\vec{a}) \times \vec{c}$

$\Rightarrow (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{c}) = (-\vec{a}) \times \vec{c}$ [by the distributive law]

$\Rightarrow (\vec{b} \times \vec{c}) + \overrightarrow{0} = \vec{c} \times \vec{a}$ $[\because \vec{c} \times \vec{c} = \overrightarrow{0}$ and $(-\vec{a}) \times \vec{c} = \vec{c} \times \vec{a}]$

$\Rightarrow \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ ...(ii)

From (i) and (ii), we get

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

OR

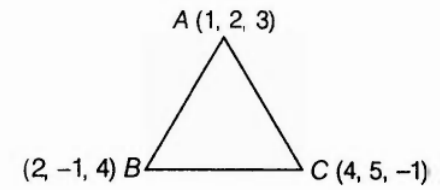
Given , ΔABC, whose vertices are A (1, 2, 3), B(2, -1, 4) and C(4, 5, -1).

Let the position vectors of the vertices A, B and C of ΔABC is

$\overrightarrow{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{OB} = 2\hat{i} - \hat{j} + 4\hat{k}$ and

$\overrightarrow{OC} = 4\hat{i} + 5\hat{j} - \hat{k}$



We know that , $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= (2\hat{i} - \hat{j} + 4\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= \hat{i} - 3\hat{j} + \hat{k}$

also , $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= (4\hat{i} + 5\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$= (3\hat{i} + 3\hat{j} - 4\hat{k})$

Now, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 1 \\ 3 & 3 & -4 \end{vmatrix}$

$= \hat{i}(12 - 3) - \hat{j}(-4 - 3) + \hat{k}(3 + 9)$

$= 9\hat{i} + 7\hat{j} + 12\hat{k}$

$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(9)^2 + (7)^2 + (12)^2}$

$= \sqrt{81 + 49 + 144}$

$= \sqrt{274}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \frac{1}{2}\sqrt{274}$

$= \frac{\sqrt{274}}{2}$

$\therefore$ $Area = \frac{\sqrt{274}}{2}$ sq units.

31. According to the question,we have to prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ if $x\sqrt{1+y} + y\sqrt{1+x} = 0$

where x≠y.

we shall first write y in terms of x explicitly i.e y=f(x)

Clearly, $x\sqrt{1+y} = -y\sqrt{1+x}$

Squaring both sides,we get,

$x^2(1 + y) = y^2 (1 + x)$

$\Rightarrow x^2 + x^2y = y^2(1 + x)$

$\Rightarrow x^2 - y^2 = y^2x - x^2y$

⇒(x - y)(x + y) = -xy(x - y)

⇒(x - y)(x + y) + xy(x - y) = 0

⇒(x - y)(x + y + xy) = 0

∴Either, x - y = 0 or x + y + xy = 0

Now, x - y = 0 ⇒ x = y

But, it is given that x ≠ y

So, it is a contradiction

Therefore, x - y = 0 is rejected.

Now, consider y + xy + x = 0

$\Rightarrow$ y(1 + x) = -x $\Rightarrow$y = $\frac{-x}{1+x}$ ....(i)

Therefore,on differentiating both sides w.r.t x, we get,

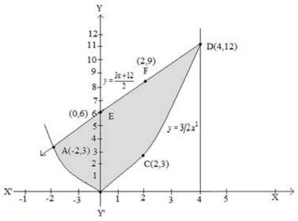
$\frac{dy}{dx} = \frac{(1+x)\times\frac{d}{dx}(-x)-(-x)\times\frac{d}{dx}(1+x)}{(1+x)^2}$[By using quotient rule of derivative]

$\Rightarrow \frac{dy}{dx} = \frac{(1+x)(-1)+x(1)}{(1+x)^2}$

$\Rightarrow \frac{dy}{dx} = \frac{-1-x+x}{(1+x)^2}$

$\therefore \frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Section D

32. 

$4y = 3x^2$ ......(1)

2y = 3x + 12......(2)

From (2), $y = \frac{3x+12}{2}$

Using this value of y in (1), we get,

x^2 - 6x - 8 = 0

$\Rightarrow$ (x + 2) (x - 4) = 0

$\Rightarrow$ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $= \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4}x^2 dx$

$= \frac{1}{2}[\frac{3x^2}{2} + 12x]_{-2}^{4} - \frac{3}{4}[\frac{x^3}{3}]_{-2}^{4}$

$\frac{1}{2}[(24+48)-(6-24)] - \frac{1}{4}[64-(-8)]$

= 45 - 18 = 27 sq units.

33. Here R is a relation on $N \times N$, defined by (a, b) R (c, d) $\Leftrightarrow$ a + d = b + c for all (a, b), (c, d) $\in N \times N$

We shall show that R satisfies the following properties

i. Reflexivity:

We know that a + b = b + a for all a, b $\in$ N.

$\therefore$ (a, b) R (a, b) for all (a, b) $\in (N \times N)$

So, R is reflexive.

ii. Symmetry:

Let (a, b) R (c, d). Then,

(a, b) R (c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R (a, b).

$\therefore$ (a, b) R (c, d) $\Rightarrow$ (c, d) R (a, b) for all (a, b), (c, d) $\in N \times N$

This shows that R is symmetric.

iii. Transitivity:

Let (a, b) R (c, d) and (c, d) R (e, f). Then,

(a, b) R (c, d) and (c, d) R (e, f)

$\Rightarrow$ a + d = b + c and c + f = d + e

$\Rightarrow$ a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R (e, f).

Thus, (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f)
This shows that R is transitive.
$\therefore$ R is reflexive, symmetric and transitive
Hence, R is an equivalence relation on $N \times N$

OR

f is one-one: For any x, y $\in$ R - {+1}, we have $f(x) = f(y)$
$\Rightarrow \frac{x}{1+|x|} = \frac{y}{|y|+1}$
$\Rightarrow$ xy + x = xy + y
$\Rightarrow$ x = y
Therefore, f is one-one function.
If f is one-one, let y = R - {1}, then f(x) = y
$\Rightarrow \frac{x}{x+1} = y$
$\Rightarrow x = \frac{y}{1-y}$
It is cleat that x $\in$ R for all y = R - {1}, also x = $\neq$ -1
Because x = -1
$\Rightarrow \frac{y}{1-y} = -1$
$\Rightarrow$ y = -1 + y
which is not possible.
Thus for each R - {1} there exists $x = \frac{y}{1-y} \in$ R - {1} such that

$$f(x) = \frac{x}{x+1} = \frac{\frac{y}{1-y}}{\frac{y}{1-y}+1} = y$$

Therefore f is onto function.

34. We have

$$A = \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$$

$\therefore$(A + A ') = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1+1 & 3+(-6) & 5+(-4) \\ -6+3 & 8+8 & 3+6 \\ -4+5 & 6+3 & 5+5 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix}$$

Suppose P = $\frac{1}{2}(A + A') = \frac{1}{2} \begin{bmatrix} 2 & -3 & 1 \\ -3 & 16 & 9 \\ 1 & 9 & 10 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix}$

And, we have (A - A') = $\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -6 & -4 \\ 3 & 8 & 6 \\ 5 & 3 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 1-1 & 3-(-6) & 5-(-4) \\ -6-3 & 8-8 & 3-6 \\ -4-5 & 6-3 & 5-5 \end{bmatrix} = \begin{bmatrix} 0 & 3+6 & 5+4 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix}$$

Let Q =$\frac{1}{2}(A - A') = \frac{1}{2} \begin{bmatrix} 0 & 9 & 9 \\ -9 & 0 & -3 \\ -9 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix}$

$$P' = \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} = \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} = P$$

$\therefore$ P is symmetric.

And $Q' = \begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-9}{2} & \frac{-9}{2} \\ \frac{9}{2} & 0 & \frac{3}{2} \\ \frac{9}{2} & \frac{-3}{2} & 0 \end{bmatrix} = -Q$

$\therefore$ Q is skew-symmetric.

Now, (P + Q) = $= \begin{bmatrix} 1 & \frac{-3}{2} & \frac{1}{2} \\ \frac{-3}{2} & 8 & \frac{9}{2} \\ \frac{1}{2} & \frac{9}{2} & 5 \end{bmatrix} + \begin{bmatrix} 0 & \frac{9}{2} & \frac{9}{2} \\ \frac{-9}{2} & 0 & \frac{-3}{2} \\ \frac{-9}{2} & \frac{3}{2} & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix} = A$

Therefore, A = P + Q

where P is symmetric and Q is skew-symmetric.

35. Let AC = x, BC = y and r be the radius of circle.

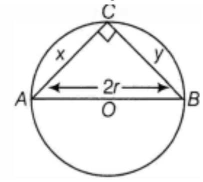
Also, $\angle C = 90^\circ$ [$\because$ angle made in semi-circle is 90°]

In ΔABC, we have

$(AB)^2 = (AC)^2 + (BC)^2$

$(2r)^2 = (x)^2 + (y)^2$

$4r^2 = x^2 + y^2$ ...(I)



We know that,

Area of ΔABC, $(A) = \frac{1}{2}\, x \cdot y$

On squaring both sides, we get

$A^2 = \frac{1}{4} x^2 y^2$

Let $A^2 = S$

Then, $S = \frac{1}{4} x^2 y^2$

$\Rightarrow \quad S = \frac{1}{4} x^2 \left(4r^2 - x^2\right)$ [from Eq. (i)]

$\Rightarrow \quad S = \frac{1}{4}\left(4x^2 r^2 - x^4\right)$

On differentiating both sides w.r.t.x, we get

$\frac{dS}{dx} = \frac{1}{4}\left(8r^2 x - 4x^3\right)$

For maxima or minima, put $\frac{dS}{dx} = 0$

$\therefore \quad \frac{1}{4}\left(8r^2 x - 4x^3\right) = 0$

$8r^2 x = 4x^3$

$8r^2 = 4x^2$

$x^2 = 2r^2$

$x = \sqrt{2}r$

From Eq. (i), we get,

$y^2 = 4r^2 - 2r^2 = 2r^2 \Rightarrow y = \sqrt{2}r$

Here, x = y so triangle is an isosceles.

Also, $\frac{d^2S}{dx^2} = \frac{d}{dx}\left[\frac{1}{4}\left(8r^2 x - 4x^3\right)\right] = \frac{1}{4}\left(8r^2 - 12x^2\right)$

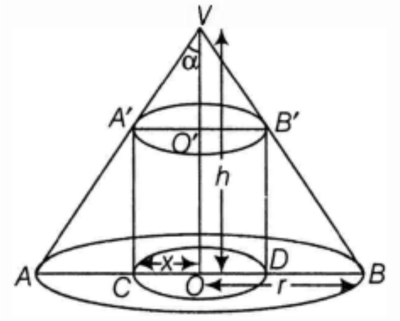
$= 2r^2 - 3x^2$ m

At $x = \sqrt{2}r, \frac{d^2S}{dx^2} = 2r^2 - 3(2r^2) = -4r^2 < 0$

Therefore, Area of the triangle is maximum when it is an isosceles triangle.

OR

Let VAB be the cone of base radius r, height h and radius of base of the inscribed cylinder be x.



Now, we observe that

$\Delta VOB \sim \Delta B'DB \Rightarrow \frac{VO}{B'D} = \frac{OB}{DB}$ [$\because$ if the triangles are similar, then their sides are proportional]

$\Rightarrow \quad \frac{h}{B'D} = \frac{r}{r-x}$

$\Rightarrow \quad B'D = \frac{h(r-x)}{r}$

Let C be the curved surface area of cylinder. Then,

C = 2π(OC)(B'D)

$\Rightarrow \quad C = \frac{2\pi x h(r-x)}{r} = \frac{2\pi h}{r}\left(rx - x^2\right)$

Therefore,on differentiating both sides w.r.t. x, we get,

$\frac{dC}{dx} = \frac{2\pi h}{r}(r - 2x)$

For Maxima or Minima, put $\frac{dC}{dx} = 0$

$\Rightarrow \quad \frac{2\pi h}{r}(r - 2x) = 0$

$\Rightarrow \quad r - 2x = 0 \Rightarrow r = 2x$

$\therefore x = \frac{r}{2}$

Therefore, radius of cylinder is half of that of cone.

Also, $\frac{d^2C}{dx^2} = \frac{d}{dx}\left[\frac{2\pi h(r-2x)}{r}\right]$

$= \frac{2\pi h}{r}(-2) = \frac{-4\pi h}{r} < 0$ as $h, r > 0$

Therefore, C is maximum or greatest.

Hence, C is greatest at x = $\frac{r}{2}$

Section E

36. i. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$

$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

ii. Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_2/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

$= \frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

iii. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

$P(E_3/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

$= \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

OR

Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

Ramesh or Ravi

$\Rightarrow P(E_2/A) + P(E_3/A) = \frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

37. i. The line along which motorcycle A is running is,

$\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, Which can be rewritten as

$(x\hat{i} + y\hat{j} + z\hat{k}) = \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k}$

$\Rightarrow x = \lambda, y = 2\lambda, z = -\lambda$

$\Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda$

Thus, the required cartesian equations is $\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$

ii. Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}, \vec{a}_2 = 3\hat{i} + 3\hat{j},\ \vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$,

$\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$

and $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$

Now, $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j})\cdot(3\hat{i} - 3\hat{j} - 3\hat{k})$

= 9 - 9 = 0

Hence, shortest distance between the given lines is 0.

iii. Since, direction cosies of a line are $\frac{k}{3}$, $\frac{k}{3}$, and $\frac{k}{3}$,

$\therefore l = \frac{k}{3}, m = \frac{k}{3}$, and $n = \frac{k}{3}$,

$\frac{k^2}{9} + \frac{k^2}{9} + \frac{k^2}{9} = 1$

$\Rightarrow \frac{3k^2}{9} = 1$

$\Rightarrow k^2 = 3$

$\Rightarrow k = \pm\sqrt{3}$

OR

The line joining the given point is:

$\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-6}{2} = \lambda$

Let $(\lambda + 1, -2\lambda + 3, 2\lambda + 6)$ be a point on the line.

Given, the points meets at XY - plane, so Z-coordinate will be zero.

$\therefore 2\lambda + 6 = 0$

$\Rightarrow \lambda = -3$

$\therefore$ Points is (-2, 9, 0)

38. i. Let number of pairs of earing = x and number of Necklaces = y

As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = Z = 30x + 40y

ii. Let number of pairs of earing = x and number of Necklaces = y

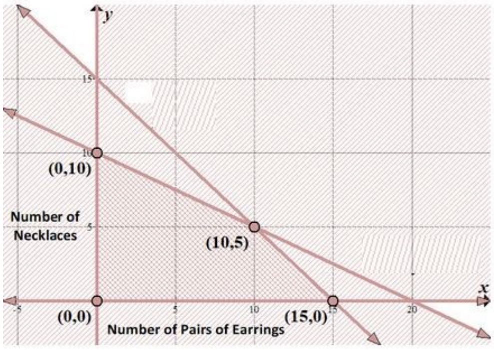
As per the given information

$x, y \geq 0$

$0.5x + y \leq 10$

$x + y \leq 15$

Profit function = Z = 30x + 40y



iii. From graph corner points are (0, 0), (0, 10), (10, 5) and (15, 0).

corner points	maximum profit = Z = 30x + 40y
(0, 0)	Z = 0
(0, 10)	Z = ₹400
(10, 5)	**Z = ₹500**
(15, 0)	Z = ₹450

Hence profit is maximum when x = number of pair of Earings = 10 and y = Number of Neckleses

OR

When x = 5 and y = 5

Z = 30x + 40y = 150 + 200 = ₹350