Class XII | Mathematics — Solutions (Set 4) | Session 2026-27

Section A

1. **(a)** reflexive and symmetric but not transitive

   Explanation:

   reflexive and symmetric but not transitive .

   Reflexivity and transitivity follows from definition.

   Here,(3,2) ,(2,1) are in R but (3,1) is not in R,so R is not transitive.

2. **(b)** $\frac{7\pi}{18}$

   Explanation:

   $\sin^{-1}(\cos\frac{\pi}{9}) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right)\right) = \sin^{-1}(\sin\frac{7\pi}{18}) = \frac{7\pi}{18}$

3. **(a)** ± 2

   Explanation:

   ± 2

   $\begin{vmatrix} 2x & -3 \\ 5 & x \end{vmatrix} = \begin{vmatrix} 10 & 1 \\ -3 & 2 \end{vmatrix}$

   $2x^2 + 15 = 20 + 3$

   $2x^2 = 23-15$

   $2x^2 = 8$

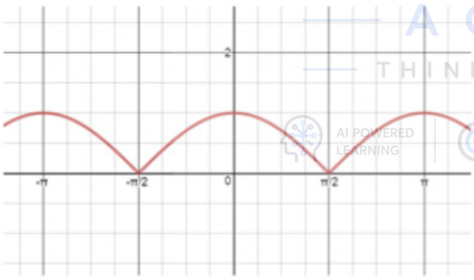
   $x^2 = 4$

   $x = \pm 2$

4. **(a)** everywhere continuous but not differentiable at $(2n + 1)\frac{\pi}{2}, n \in Z$

   Explanation:

   Given that $f(x) = |\cos x|$

   From the graph it is evident that it is everywhere continuous but not differentiable at $(2n + 1)\frac{\pi}{2}, n \in Z$

5. **(c)** $\frac{\pi}{2}$

   Explanation:

   $\frac{\pi}{2}$

6. **(b)** Both (i) and (ii)

   Explanation:

   Both (i) and (ii)

7. **(b)** half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.

Explanation:

The inequality 2x + 3y > 6 represents a half-plane where points satisfy the condition of being above the line 2x + 3y = 6. The line 2x + 3y = 6 is not included in the solution, since the inequality is strict (">" and not "geq").

To determine which half-plane, we can test a point not on the line. Testing the origin (0, 0) : Substitute x = 0 and y = 0 into the inequality: 2(0) + 3(0) = 0 Since $0 \ngtr 6$, the origin is not in the solution region.

Thus, the solution is a half-plane that does not contain the origin nor the points on the line.

The correct option is: half-plane that neither contains the origin nor the points of the line 2x + 3y = 6.

8.

(c) $\sqrt{62}$

Explanation:

We have:

$\vec{a} = 2\hat{i} - 7\hat{j} - 3\hat{k}$,

then,

$\left|\overrightarrow{a}\right| = \sqrt{2^2 + (-7)^2 + (-3)^2} = \sqrt{4 + 49 + 9} = \sqrt{62}$.

9.

(d) $-\frac{1}{3}\log|2 - 3x| + C$

Explanation:

Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{x^2} dx = \log x + c$

Therefore ,

Put 2 - 3 x = t

-3 dx = dt

$= \frac{1}{3} \int \frac{1}{t} dt$

$= -\frac{1}{3}\log t + c$

$= -\frac{1}{3}\log|2 - 3x| + c$

10. **(a)** 2×3

Explanation:

Order of a matrix is given by

(number of rows) $\times$ (number of columns)

$\therefore$ Order of matrix A = 2 $\times$ 3

11.

(d) (40,15)

Explanation:

We need to maximize the function z = x + y Converting the given inequations into equations, we obtain

x + 2y = 70, 2x + y = 95, x = 0 and y = 0

Region represented by x + 2y $\leq$ 70 :

The line x + 2y = 70 meets the coordinate axes at A(70, 0) and B(0, 35) respectively. By joining these points we obtain the line x + 2y = 70. Clearly (0, 0) satisfies the inequation x + 2y $\leq$ 70 . So, the region containing the origin represents the solution set of the inequation x + 2y $\leq$ 70.

Region represented by 2x + y $\leq$ 95 :

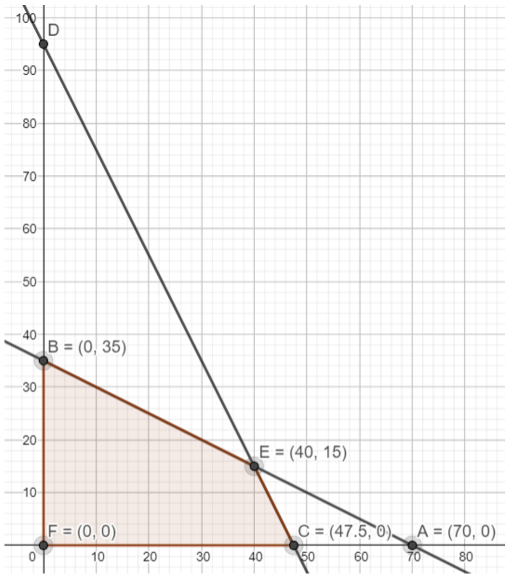
The line 2x + y = 95 meets the coordinate axes at $C\left(\frac{95}{2}, 0\right)$ respectively. By joining these points we obtain the line 2x + y = 95

Clearly (0, 0) satisfies the inequation 2x + y $\leq$ 95 . So, the region containing the origin represents the solution set of the inequation 2x + y $\leq$ 95

Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0

The feasible region determined by the system of constraints x + 2y $\leq$ 70, 2x + y $\leq$ 95, x $\geq$ 0, and y $\geq$ 0 are as follows.



The corner points of the feasible region are O(0, 0), C($\frac{95}{2}$, 0) E(40, 15) and B(0, 35).

The value fo Z at these corner points are as follows.

Corner point : z = x + y

O(0, 0) : 0 + 0 = 0

$C\left(\frac{95}{2}, 0\right) : \frac{95}{2} + 0 = \frac{95}{2}$

E(40, 15) : 40 + 15 = 55

B(0, 35) : 0 + 35 = 35

We see that maximum value of the objective function Z is 55 which is at (40, 15).

12.

(b) -1

Explanation:

-1

13.

(b) 0

Explanation:

By evaluating given determinant and using sin (90 - A) = cos A. we get value of det. = 0

14. **(a)** $\frac{4}{7}$

Explanation:

Let E_1 = Event that first ball is red = (RRR, RRB, RBR, RBB)

And E_2 = Event that exactly two of three balls being red = (RRR, RRB)

$\therefore P\left(E_1\right) = P_R \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_B + P_R \cdot P_B \cdot P_R + P_R \cdot P_B \cdot P_B$

$= \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{2}{6}$

$= \frac{60+60+60+30}{336} = \frac{210}{336}$

$P\left(E_1 \cap E_2\right) = P_B \cdot P_R \cdot P_R + P_R \cdot P_R \cdot P_R$

$= \frac{5}{8} \cdot \frac{3}{7} \cdot \frac{4}{6} + \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{120}{336}$

$\therefore P\left(E_2 / E_1\right) = \frac{P\left(E_1 \cap E_2\right)}{P\left(E_1\right)} = \frac{120/336}{210/336} = \frac{4}{7}$

15. **(a)** $y = e^{cx}$

Explanation:

$y \log y \, dx = x \, dy$

$\int \frac{1}{x} dx = \int \frac{1}{y \log y} dy$

$\log|x| = \log|\log y| + \log C$ Since $\int \frac{f'(x) dx}{f(x)} = log|f(x)| + c$ and $\frac{1}{C} = c \, a \, new \, constant$

$\log x = \log(C \log y)$

$x = C \log y$
$\log y = \frac{1}{C} x$
$\log y = cx$
$y = e^{cx}$

16.

(b) symmetric

Explanation:

Let (x,y)∈R , such that x⊥y.

We can also write from above that, y⊥x.

Hence, (y,x)∈R

So, They are symmetric.

17.

(d) No value

Explanation:

No value

18. **(a)** perpendicular to z-axis

Explanation:

We have,

$\frac{x-3}{3} = \frac{y-2}{1} = \frac{z-1}{0}$

Also, the given line is parallel to the vector $\vec{b} = 3\hat{i} + \hat{j} + 0\hat{k}$

Let $x\hat{i} + y\hat{j} + z\hat{k}$ be perpendicular to the given line.

Now,

3x + 4y + 0z = 0

It is satisfied by the coordinates of z-axis, i.e. (0, 0, 1)

Hence, the given line is perpendicular to z-axis.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both A and R are true and R is the correct explanation of A.

20. **(a)** 8

Explanation:

We know that, dot product of two orthogonal vectors is always 0.

Hence,

$(2\hat{i} - \hat{j} + 2\hat{k})$. $(3\hat{i} + \lambda\hat{j} + \hat{k})$ = 0

⇒ 6 - λ + 2 = 0

⇒ λ = 8

Section B

21. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \tan^{-1}\sqrt{3} - [\pi - \sec^{-1}2]$

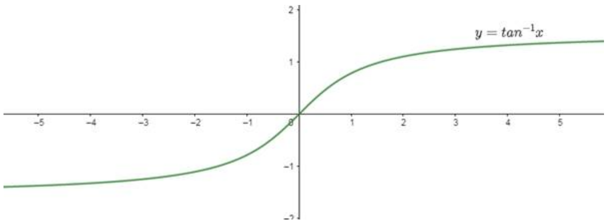
$= \frac{\pi}{3} - \pi + \cos^{-1}\left(\frac{1}{2}\right)$

$= -\frac{2\pi}{3} + \frac{\pi}{3} = -\frac{\pi}{3}$

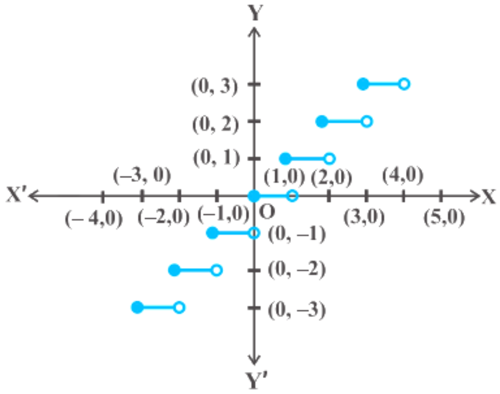
OR

Principal value branch of $\tan^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

and its graph is shown below.



22. First, observe that f is defined for all real numbers. The graph of the function is given in the figure. From the graph, it looks like that f is discontinuous at every integral point. Below we explore if this is true.



Case 1: Let c be a real number which is not equal to any integer. It is evident from the graph that for all real numbers close to c the value of the function is equal to [c]; i.e., $\lim_{x\to c} f(x) = \lim_{x\to c} [x] = [c]$. Also f(c) = [c] and hence the function is continuous at all real numbers not equal to integers.

Case 2: Let c be an integer. Then we can find a sufficiently small real number r > 0 such that [c – r] = c – 1 whereas [c + r] = c. This, in terms of limits mean that

$\lim_{x\to c^-} f(x) = c - 1, \lim_{x\to c^+} f(x) = c$

Since these limits cannot be equal to each other for any c, the function is discontinuous at every integral point.

23. Given: $f(x) = (x + 2)e^{-x}$

$f'(x) = e^{-x} - e^{-x}(x+2)$

$= e^{-x}(1 - x - 2)$

$= -e^{-x}(x+1)$

For Critical points

$f'(x) = 0$

$\Rightarrow -e^{-x}(x + 1) = 0$

$\Rightarrow x = -1$

Clearly $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence f(x) increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

OR

Taking log of both sides, we get

$30 \log x + 20 \log y = 50 \log(x + y)$

Differentiating both sides w.r.t. x, we get

$\frac{30}{x} + \frac{20}{y}\frac{dy}{dx} = \frac{50}{x+y}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \frac{dy}{dx}\left(\frac{20x-30y}{y(x+y)}\right) = \frac{20x-30y}{x(x+y)}$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

24. Let $I = \int e^x (\cot x - \operatorname{cosec}^2 x)\, dx$

Here, $f(x) = \cot x$ put $e^x f(x) = t$

$f'(x) = -\operatorname{cosec}^2 x$

let $e^x \cot x = t$

Diff. both sides w.r.t x

$e^x \cot x + e^x(-\operatorname{cosec}^2 x) = \frac{dt}{dx}$

$\Rightarrow e^x(\cot x - \operatorname{cosec}^2 x) = dt$

$\therefore \int e^x (\cot x - \operatorname{cosec}^2 dx)\, dx$

$= t + C\ [e^x \cot x = t]$

$= e^x \cot x + C$

25. Given:

$$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

To find the matrix as a result of the product AA^T

Firstly, we find the A^T(which is the transpose of the matrix A)

If $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is a 2×2 matrix, then the transpose of a matrix is $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$

So,

$$A^T = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore AA^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$ (Using the rule of matrix multiplication we get)

$$= \begin{bmatrix} \cos x \times \cos x + (-\sin x) \times (-\sin x) & \sin x \times \cos x + (-\sin x) \times \cos x \\ \sin x \times \cos x + \cos x \times (-\sin x) & \sin x \times \sin x + \cos x \times \cos x \end{bmatrix}$$

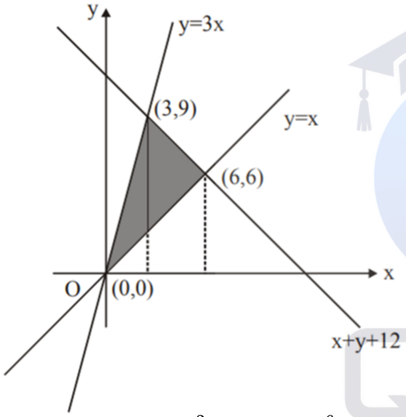
$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin x \cos x - \sin x \cos x \\ \sin x \cos x - \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix}$$

$AA^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ [$\because \cos^2 x + \sin^2 x = 1$ using the property of the trigonometry identity.]

$\Rightarrow AA^T = I_2$

Section C

26.



Required area = $\int_0^3 3x dx + \int_3^6 (12 - x)dx - \int_0^6 x dx$

$= 3\left[\frac{x^2}{2}\right]_0^3 + \left[12x - \frac{x^2}{2}\right]_3^6 - \left[\frac{x^2}{2}\right]_0^6$

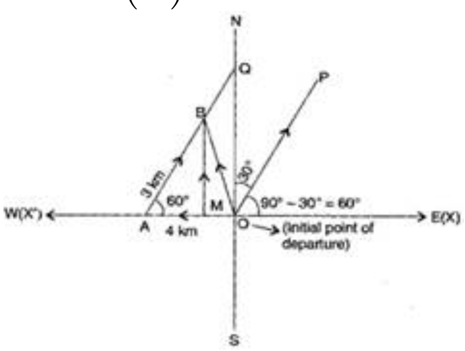
$= \frac{27}{2} + \frac{45}{2} - 18 = 18$ sq units

27. Let the initial point of departure is origin (0, 0) and the girl walks a distance OA = 4 km towards west.

Through the point A, draw a line AQ parallel to a line OP, which is 30^0 East of North, i.e., in East-North quadrant making an angle of 30^0 with North.

Again, let the girl walks a distance AB = 3 km along this direction $\overrightarrow{OQ}$

$\therefore \overrightarrow{OA} = 4\left(-\vec{i}\right) = -4\hat{i}$ ...(i) [$\because$ Vector $\overrightarrow{OA}$ is along OX']



Now, draw BM perpendicular to x - axis.

In ΔAMB by Triangle Law of Addition of vectors,

$\overrightarrow{AB} = \overrightarrow{AM} + \overrightarrow{MB} = (AM)\,\hat{i} + (MB)\,\hat{i}$

Dividing and multiplying by AB in R.H.S.,

$\overrightarrow{AB} = AB\frac{AM}{AB}\hat{i} + AB\frac{MB}{AB}\hat{j} = 3\cos 60^o\hat{i} + 3\sin 60^o\hat{j}$

$\Rightarrow AB = 3\frac{1}{2}\hat{i} + 3\frac{\sqrt{3}}{2}\hat{i} = \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}j$ ...(ii)

$\therefore$ Girl's displacement from her initial point O of departure to final point B,

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = -4\hat{i} + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}\right) = \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$\Rightarrow \overrightarrow{OB} = \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

28. Let

$I = \int_0^x \frac{1}{1+\sin x}dx$

Multiplying Numerator and Denominator of the integrand by (1-sin x), gives

$I = \int_0^x \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}dx$

$= \int_0^x \frac{(1-\sin x)}{(1^2-\sin^2 x)}dx$

$= \int_0^x \frac{1-\sin x}{(\cos^2 x)}dx$

$= \int_0^x \frac{1}{\cos^2 x}dx - \int_0^\pi \frac{\sin x}{\cos^2 x}dx$

$= \int_0^x \sec^2 x dx - \int_0^x \tan x \cdot \sec x dx$

$I = [\tan x]_0^x - [\sec x]_0^x$

$= [\tan\pi - \tan 0] - [\sec\pi - \sec 0]$

= [0 - 0] - [-1 - 1]

= 2

$\therefore \int_0^x \frac{1}{1+\sin x}dx = 2$

OR

Let the given integral be,

I = $\int \frac{1}{5-4\cos x}dx$

Putting cos x = $\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\Rightarrow I = \int \frac{1}{5-4\left(\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$

$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{5\left(1+\tan^2\frac{x}{2}\right)-4+4\tan^2\frac{x}{2}}dx$

$= \int \frac{\sec^2\left(\frac{x}{2}\right)}{9\tan^2\frac{x}{2}+1}dx$

Let tan $\left(\frac{x}{2}\right)$ = t

$\Rightarrow \frac{1}{2}$ sec^2 $\left(\frac{x}{2}\right)$ dx = dt

sec^2 $\left(\frac{x}{2}\right)$ dx = 2dt

$\therefore I = \int \frac{2dt}{1-t^2-2t}$

$= 2\int \frac{-2dt}{t^2+2t-1}$

$= 2\int \frac{-2dt}{t^2+2t+1-2}$

$= \frac{2}{9} \times 3\tan^{-1}\left(\frac{t}{\frac{1}{3}}\right) + C$

$= \frac{2}{3}\tan^{-1}(3t) + C$

$= \frac{2}{3}\tan^{-1}\left(3\tan\frac{x}{2}\right) + C$

29. The given differential equation is,

$(x^2 + 1)\frac{dy}{dx}$ - 2xy = $(x^2 + 1)(x^2 + 2)$

$\Rightarrow \frac{dy}{dx} + \left(\frac{-2x}{x^2+1}\right)y = x^2 + 2$

This is of the form $\frac{dy}{dx} + Py = Q$, where

$P = \frac{-2x}{x^2+1}$ and Q = $x^2 + 2$

Thus the given differential equation is linear differential equation

Now, $IF = e^{\int P\,dx}$

$= e^{\int \frac{-2x}{x^2+1} dx}$

$= e^{-\log\left(x^2+1\right)} = \left(x^2+1\right)^{-1} = \frac{1}{x^2+1}$

Therefore the solution is given by

$(IF) \cdot y = \int (IF)Q + C$

$\Rightarrow \frac{1}{x^2+1} \cdot y = \int \frac{1}{x^2+1}\left(x^2+2\right) dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \frac{x^2+2}{x^2+1} dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \frac{x^2+1+1}{x^2+1} dx + C$

$\Rightarrow \frac{y}{x^2+1} = \int \left\{1 + \frac{1}{x^2+1}\right\} dx + C$

$\Rightarrow \frac{y}{x^2+1} = x + \tan^{-1} x + C$

$\Rightarrow$ y = $(x^2 + 1)(x + \tan^{-1}x + C)$

OR

The given differential equation is,

$\left[x\sqrt{x^2+y^2} - y^2\right]$ dx + xy dy = 0

$\frac{dy}{dx} = \frac{y^2 - x\sqrt{x^2+y^2}}{xy}$

This is a homogeneous differential equation

Putting y = vx and $\frac{dy}{dx}$ = v + x$\frac{dv}{dx}$, we get

$v + x\frac{dv}{dx} = \frac{v^2x^2 - x\sqrt{x^2+v^2x^2}}{vx^2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{v^2 - \sqrt{1+v^2}}{v}$

$\Rightarrow v + x\frac{dv}{dx} = v - \frac{\sqrt{1+v^2}}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{-\sqrt{1+v^2}}{v}$

$\Rightarrow \frac{v}{\sqrt{1+v^2}} dv = -\frac{1}{x} dx$

Putting 1 + v^2 = t, we get

v dv = $\frac{dt}{2}$

$\therefore \frac{1}{2\sqrt{t}} dt = -\frac{1}{x} dx$

Integrating both sides, we get

$\int \frac{1}{2\sqrt{t}} dt = -\int \frac{1}{x} dx$

$\Rightarrow \sqrt{t}$ = -log |x| + log C ...(i)

Substituting the value of t in (i), we get

$\sqrt{1+v^2} = \log\left|\frac{C}{x}\right|$

Hence, $\sqrt{y^2+x^2} = x\log\left|\frac{C}{x}\right|$ is the required solution.

30. We need to maximize z = x + y

First, we will convert the given inequations into equations, we obtain the following equations:

-2x + y = 1, x = 2, x + y = 3, x = 0 and y = 0

The line -2x + y = 1 meets the coordinate axis at $A\left(\frac{-1}{2}, 0\right)$ and B(0, 1). Join these points to obtain the line -2x + y = 1.

Clearly, (0, 0) satisfies the inequation -2x + y $\leq$ 1. So, the region in xy-plane that contains the origin represents the solution set of the given equation.

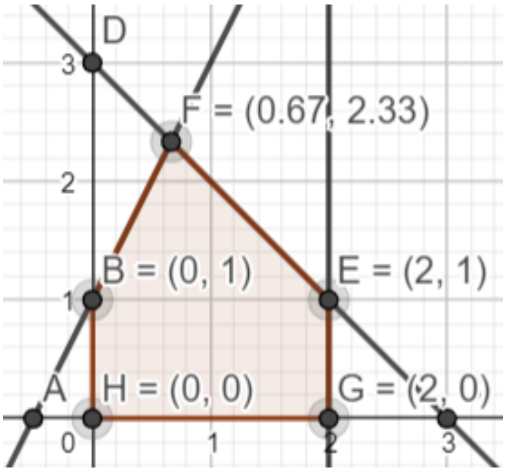
x = 2 is the line passing through (2,0) and parallel to the Y axis.

The region below the line x = 2 will satisfy the given inequation. The line x + y = 3 meets the coordinate axis at C(3, 0) and D(0, 3). Join these points to obtain the line x + y = 3.

Clearly, (0,0) satisfies the inequation x + y $\leq$ 3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

Region represented by x $\geq$ 0 and y $\geq$ 0 (non -negative restrictions)

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the regior represented by the inequations. These lines are drawn using a suitable scale.



The corner points of the feasible region are O(0,0), G(2,0), E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

The values of objective function at the corner points are as follows:

Corner point : Z = x + y

O(0, 0) : 0 + 0 = 0

C(2, 0) : 2 + 0=2

E(2, 1) : 2 + 1=3

$F\left(\frac{2}{3}, \frac{7}{3}\right) : \frac{2}{3}+\frac{7}{3}=\frac{9}{3}=3$

We see that the maximum value of the objective function z is 3 which is at E(2,1) and $F\left(\frac{2}{3}, \frac{7}{3}\right)$

Thus, the optimal value of objective function z is 3.

OR

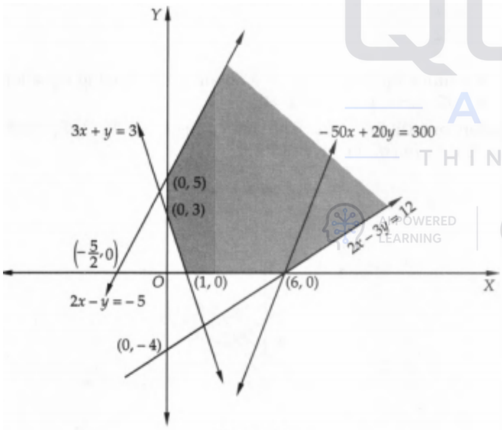
2x - y $\geq$ - 5

3x + y $\geq$ 3

2x - 3y $\leq$ 12

x $\geq$ 0, y $\geq$ 0

The feasible region of the system of inequations given in constraints is shown in a figure. We observe that the feasible region is unbounded.



The values of the objective function Z at the comer points are given in the following table:

Corner point (x, y)	Value of the objective function Z = -50x + 20y
(0,5)	Z = - 50 $\times$ 0 + 20 $\times$ 5 = 100
(0,3)	Z = - 50 $\times$ 0 + 20 $\times$ 3 = 60
(1,0)	Z = - 50 $\times$ 1 + 20 $\times$ 0 = - 50
(6,0)	Z = -50 $\times$ 6 + 20 $\times$ 0 = - 300

Clearly, - 300 is the smallest value of Z at the corner point (6, 0). Since the feasible region is unbounded, therefore, to check whether - 300 is the minimum value of Z, we draw the line - 300 = -50x + 20y and check whether the open half plane -50 x + 20y < -300 has points in common with the feasible region or not. From Fig., we find that the open half plane represented by - 50 x + 20y < - 300 has points in common with the feasible region. Therefore, Z = - 50x + 20y has no minimum value subject to the given constraints.

31. To show that the given function is continuous at x = 0, we show that

$(LHL)_{x=0} = (RHL)_{x=0} = f(0)$ ....(i)

Here, we have $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x > 0 \\ 2, & x = 0 \\ \frac{4(1-\sqrt{1-x})}{x}, & x < 0 \end{cases}$

Now, LHL = $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{4(1-\sqrt{1-x})}{x}$

$= \lim_{h \to 0} \frac{4[1-\sqrt{1-(0-h)}]}{0-h}$

$= \lim_{h \to 0} \frac{4[1-\sqrt{1+h}]}{-h}$

$= \lim_{h \to 0} \frac{4[1-\sqrt{1+h}]}{-h} \times \frac{1+\sqrt{1+h}}{1+\sqrt{1+h}}$

$= \lim_{h \to 0} \frac{4\left[(1)^2-(\sqrt{1+h})^2\right]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h \to 0} \frac{4[1-(1+h)]}{-h[1+\sqrt{1+h}]}$

$= \lim_{h \to 0} \frac{-h \times 4}{-h[1+\sqrt{1+h}]}$

$= \lim_{h \to 0} \frac{4}{1+\sqrt{1+h}}$

$= \frac{4}{1+\sqrt{1}} = \frac{4}{2}$ = 2

and RHL = $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left(\frac{\sin x}{x} + \cos x\right)$

$\Rightarrow \quad \text{RHL} = \lim_{h \to 0} \left(\frac{\sin h}{h} + \cos h\right)$

$= \lim_{h \to 0} \frac{\sin h}{h} + \lim_{h \to 0} \cos h$

= 1 + cos 0

= 1 + 1

= 2

Also , given that x = 0, f(x) = 2 $\Rightarrow$ f(0) = 2

Since, $(LHL)_{x=0} = (RHL)_{x=0} = f(0) = 2$

Therefore, f(x) is continuous at x = 0.

Section D

32. We have

$\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}$, where $x^2 = t$

$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$

Use partial fractions for 2nd part

Let $\frac{(4t+10)}{(t+3)(t+4)} = \frac{A}{(t+3)} + \frac{B}{(t+4)}$

$\Rightarrow$ (4t +10) = A(t + 4) + B(t + 3) .... (i)

Putting t = -3 in (i), we get A = -2

Putting t = -4 in (i), we get B = 6

$\therefore \frac{(4t+10)}{(t+3)(t+4)} = \frac{-2}{(t+3)} + \frac{6}{(t+4)}$ ..... (ii)

Thus, $\frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} = \frac{(t+1)(t+2)}{(t+3)(t+4)}$, where $x^2 = t$

$= \frac{(t^2+3t+2)}{(t^2+7t+12)} = 1 - \frac{(4t+10)}{(t+3)(t+4)}$

$= 1 - \left\{\frac{-2}{(t+3)} + \frac{6}{(t+4)}\right\}$ [from (ii)]

$= \left\{1 + \frac{2}{(t+3)} - \frac{6}{(t+4)}\right\}$

$= \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\}$

$\therefore \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx = \int \left\{1 + \frac{2}{(x^2+3)} - \frac{6}{(x^2+4)}\right\} dx$

$= \int dx + 2\int \frac{dx}{(x^2+3)} - 6\int \frac{dx}{(x^2+4)}$

$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{6}{2} \tan^{-1}\left(\frac{x}{2}\right) + C$

$= x + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - 3\tan^{-1}\left(\frac{x}{2}\right) + C$

33. Given that, A = R - {3}, B = R - {1}.

$f : A \to B$ is defined by $f(x) = \frac{x-2}{x-3}$ $\forall x \in A$

For injectivity

Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$\Rightarrow (x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -x_1 = -x_2 \Rightarrow x_1 = x_2$

So, f(x) is an injective function

For surjectivity

Let $y = \frac{x-2}{x-3} \Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = 2 - 3y $\Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A,\ \forall y \in B$ [codomain]

So, f(x) is surjective function.

Hence, f(x) is a bijective function.

OR

Given that A = {1, 2, 3, ...9} (a, b) R (c, d) a + d = b + c for (a, b) $\in$ A$\times$A and (c, d) $\in$ A$\times$A.

Let (a, b) R (a, b)

$\Rightarrow$ a + b = b + a, $\forall$a, b $\in$ A

Which is true for any a, b $\in$ A

Hence, R is reflexive.

Let (a, b) R (c, d)

a+d = b+c

c + b = d + a $\Rightarrow$ (c, d) R (a, b)

So, R is symmetric.

Let (a, b) R (c, d) and (c, d) R (e, f)

a + d = b + c and c + f = d + e

a + d = b + c and d + e = c + f(a + d) - (d + e) = (b + c) - (c + f)

(a - e) = b - f

a + f = b + e

(a, b) R (e, f)

So, R is transitive.

Hence R is an equivalence relation.

Let (a,b) R (2,5),then

a+5=b+2

a=b-3

If b<3 ,then a does not belong to A.

Therefore,possible values of b are >3.

For b=4,5,6,7,8,9

a=1,2,3,4,5,6

Therefore, equivalence class of (2,5) is

{(1,4),(2,5),(3,6),(4,7),(5,8),(6,9).

34. We have, $A = \begin{vmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{vmatrix}$ ...(i)

$\therefore |A| = 1(-3) - 2(-2) + 0 = 1 \neq 0$

Now, $A_{11} = -3, A_{12} = 2, A_{13} = 2,\ A_{21} = -2, A_{22} = 1, A_{23} = 1,\ A_{31} = -4, A_{32} = 2$ and $A_{33} = 3$

$\therefore adj\,(A) = \begin{vmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{vmatrix}^T = \begin{vmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{vmatrix}$

$\therefore A^{-1} = \frac{adjA}{|A|}$

$= \frac{1}{1}\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$ ...(i)

Also, we have the system of linear equations as

$x - 2y = 10$

$2x - y - z = 8$

and $-2y + z = 7$

Now, the given system of equations can be rewritten in the form AX=B,

where, $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

Since A is non singular , therefore given system of equations has a unique solution given by,

$\therefore X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$

$= \begin{bmatrix} -30 + 60 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

$\therefore x = 0, y = -5$ and $z = -3$

35. Given Cartesian equations of lines

$\mathrm{L}_1 = \frac{\mathrm{x}-1}{3} = \frac{\mathrm{y}+1}{2} = \frac{\mathrm{z}-1}{5}$

Line L1 is passing through point (1, -1, 1) and has direction ratios (3, 2, 5)

Thus, vector equation of line L1 is

$\vec{\mathrm{r}} = (\hat{\mathrm{i}} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

And

$\mathrm{L}_2 : \frac{\mathrm{x}-2}{2} = \frac{\mathrm{y}-1}{3} = \frac{\mathrm{z}+1}{-2}$

Line L_2 is passing through point (2, -1, 1) and has direction ratios (2, 3, -2)

Thus, vector equation of line L2 is

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Now, to calculate distance between the lines,

$\vec{\mathrm{r}} = (\hat{\imath} - \hat{\mathrm{j}} + \hat{\mathrm{k}}) + \lambda(3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}})$

$\vec{\mathbf{r}} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}) + \mu(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}})$

Here,we have

$\overrightarrow{a_1} = 1 - j + \hat{k}$

$\overrightarrow{\mathrm{b}_1} = 3\hat{\mathrm{i}} + 2\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$

$\overrightarrow{\mathrm{b}_2} = 2\hat{\mathrm{i}} + 3\hat{\mathrm{j}} - 2\hat{\mathrm{k}}$

Thus,

$\overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{\mathrm{k}} \\ 3 & 2 & 5 \\ 2 & 3 & -2 \end{vmatrix}$

$= \hat{\imath}(-4 - 15) - \hat{\jmath}(-6 - 10) + \hat{k}(9 - 4)$

$\Rightarrow \overrightarrow{\mathrm{b}_1} \times \overrightarrow{\mathrm{b}_2} = -19\hat{\mathrm{i}} + 16\hat{\mathrm{j}} + 5\hat{\mathrm{k}}$

$\Rightarrow |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{(-19)^2 + 16^2 + 5^2}$
$= \sqrt{361 + 256 + 25}$
$= \sqrt{642}$
$\overrightarrow{a_2} - \overrightarrow{a_1} = (2-1)\hat{i} + (1+1)\hat{j} + (-1-1)\hat{k}$
$\therefore \overrightarrow{a_2} - \overrightarrow{a_1} = 1 + 2\hat{j} - 2\hat{k}$
Now,
$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = (-19\hat{i} + 16\hat{j} + 5\hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$
$= ((-19) \times 1) + (16 \times 2) + (5 \times (-2))$
= -19 + 32 - 10
= 3
Thus, the shortest distance between the given lines is
$d = \left| \frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} \right|$
$\Rightarrow d = \left| \frac{3}{\sqrt{642}} \right|$
$\therefore d = \frac{3}{\sqrt{642}}$ units
As $d \neq 0$
Hence, given lines do not intersect each other.

OR

Here,it is given that the equation of lines
$L1 : \frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$
$L2 = \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$
Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.
Suppose general point on line L_1 is P = (x_1, y_1, z_1)
$x_1 = 3s + 3$, $y_1 = -s + 8$, $z_1 = s + 3$
and suppose general point on line L_2 is Q = (x_2, y_2, z_2)
$x_2 = -3t - 3$, $y_2 = 2t - 7$, $Z_2 = 4t + 6$
$\therefore \overrightarrow{PQ} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$
$= (-3t - 3 - 3s - 3)\hat{i} + (2t - 7 + s - 8)j + (4t + 6 - s - 3)\hat{k}$
$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\hat{i} + (2t + s - 15)j + (4t - s + 3)\hat{k}$
Direction ratios of $\overrightarrow{PQ}$ are ((-3t - 3s - 6, (2t + s - 15), (4t - s + 3))
PQ will be the shortest distance if it perpendicular to both the given lines
Thus, by the condition of perpendicularity,
$\Longrightarrow$ 3(-3t - 3s - 6) - 1(2t + s - 15) + 1(4t - s + 3) = 0 and
$\Longrightarrow$ -3(-3t - 3s - 6) + 2(2t + s - 15) + 4(4t - s + 3) = 0
$\Rightarrow$ -9t - 9s - 18 -2t - s + 15 + 4t - s + 3 = 0 and
9t + 9s + 18 + 4t + 2s - 30 + 16t - 4s + 12 = 0
$\Rightarrow$ -7t - 11s = 0 and
29t + 7s = 0
Solving above two equations, we get,
t = 0 and s = 0
therefore,
P = (3, 8, 3) and Q = (-3, -7, 6)
Now distance between points P and Q is
$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$
$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$
$= \sqrt{36 + 225 + 9}$
$= \sqrt{270}$
$= 3\sqrt{30}$
Thus, the shortest distance between two given lines is

$d = 3\sqrt{30}$ units

Now, equation of line passing through points P and Q is,

$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$

$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$

$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$

$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Thus, equation of line of shortest distance between two given lines is

$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Section E

36. i. P(A) = $\frac{1}{3}$, P(A') = 1 $- \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = 1 $- \frac{1}{3} = \frac{1}{2}$

P(Both are selected) = $P(A \cap B) = P(A) \cdot P(B)$ = $\frac{1}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{1}{6}$

ii. P(A) = $\frac{1}{3}$, P(A') = 1 $- \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = 1 $- \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A' \cap B') = P(A') \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{1}{3}$

iii. P(A) = $\frac{1}{3}$, P(A') = 1 $- \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = 1 $- \frac{1}{3} = \frac{1}{2}$

P(none of them selected) = $P(A') \cdot P(B) + P(A) \cdot P(B')$ = $\frac{2}{3} . \frac{1}{2} + \frac{1}{3} . \frac{1}{2}$

P(Both are selected) = $\frac{3}{6} = \frac{1}{2}$

OR

P(A) = $\frac{1}{3}$, P(A') = 1 $- \frac{1}{3} = \frac{2}{3}$

P(B) = $\frac{1}{2}$, P(b') = 1 $- \frac{1}{3} = \frac{1}{2}$

P(atleast one of them selected) = 1 - P(none selected) = 1 $- \frac{1}{3}$

P(atleast one of them selected) = $\frac{2}{3}$

37. i. If ₹ 15000 is invested in bond X, then the amount invested in bond Y = ₹ (35000 - 15000) = ₹ 20000.

A = Investment $\begin{bmatrix} X & Y \\ 15000 & 20000 \end{bmatrix}$

and B = $\begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \textit{Interest rate} \\ \begin{bmatrix} 10\% \\ 8\% \end{bmatrix} \end{array} = \begin{array}{c} \\ X \\ Y \end{array} \begin{array}{c} \textit{Interest rate} \\ \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} \end{array}$

ii. The amount of interest received on each bond is given by

AB = $\begin{bmatrix} 15000 & 20000 \end{bmatrix} \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$

= [15000 $\times$ 0.1 + 20000 $\times$ 0.08] = [1500 + 1600] = 3100

iii. Let ₹ x be invested in bond X and then ₹ (35000 - x) will be invested in bond Y.

Now, total amount of interest is given by

$\begin{bmatrix} x & 35000 - x \end{bmatrix} \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$ = [0.1x + (35000 - x)0.08]

But, it is given that total amount of interest = ₹ 3200

$\therefore$ 0.1x + 2800 - 0.08x = 3200

$\Rightarrow$ 0.02x = 400 $\Rightarrow$ x = 20000

Thus, ₹ 20000 invested in bond X and ₹ 35000 - ₹ 20000 = ₹ 15000 invested in bond Y.

OR

Let ₹ x invested in bond X, then we have

$x \times \frac{10}{100}$ = 500 $\Rightarrow$ x = 5000

Thus, amount invested in bond X is ₹ 5000 and so investment in bond Y be ₹ (35000 - 5000) = ₹ 30000

38. i. Since 'C' is cost of making tank

$\therefore$ C = 70xy + 45 $\times$ 2(2x + 2y)

$\Rightarrow$ C = 70xy + 90(2x + 2y)

$\Rightarrow$ C = 70xy + 180(x + y) $[\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}]$

$\Rightarrow$ C = 70x $\times \frac{4}{x} + 180\left(x + \frac{4}{x}\right)$

$\Rightarrow$ C = 280 + 180$\left(x + \frac{4}{x}\right)$

ii. x·y = 4

Volume of tank = length $\times$ breadth $\times$ height (Depth)

8 = $x \cdot y \cdot$ 2

$\Rightarrow$ 2xy = 8 $\Rightarrow$ xy = 4

iii. For maximum or minimum

$\frac{dC}{dx} = 0$

$\frac{d}{dx}(280 + 180(x + \frac{4}{x})) = 0 \Rightarrow 180\left(1 + 4\left(-\frac{1}{x^2}\right)\right) = 0$

$\Rightarrow 180\left(1 - \frac{4}{x^2}\right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$

$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2$

$\Rightarrow$ x = 2 (length can never be negative)

OR

Now, $\frac{d^2C}{dx^2} = 180\left(+\frac{8}{x^3}\right)$

$\Rightarrow \left.\frac{d^2C}{dx^2}\right|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$

Hence, to minimize C, x = 2m