

Section A

1.

(b) $\begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$

Explanation:

Let $B = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{vmatrix} 1 & -2 \\ 1 & -4 \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

$$\begin{vmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{vmatrix} = \begin{vmatrix} 6 & 0 \\ 0 & 6 \end{vmatrix}$$

Now, comparing the corresponding element

$$a + b = 6 \dots (i)$$

$$-2a + 4b = 0$$

$$-2a = -4b$$

$$a = 2b \dots (ii)$$

$$c + d = 0 \dots (iii)$$

$$-2c + 4d = 6$$

putting (ii) in (i)

$$3b = 6$$

$$b = 2$$

$$\therefore a = 4$$

Now, from equation (iii) and (iv)

$$-2c + 4(-c) = 6$$

$$-6c = 6$$

$$c = -1$$

$$\therefore d = 1$$

$$\text{matrix } B = \begin{vmatrix} 4 & 2 \\ -1 & 1 \end{vmatrix}$$

2.

(c) $\frac{1}{19}$

Explanation:

$$A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$$

Using adjoint matrix

$$A^{-1} = \frac{-1}{19} \begin{bmatrix} -2 & -3 \\ -5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{19} \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix} = \frac{1}{19} A$$

$$k = \frac{1}{19}$$

3.

(a) 0

Explanation:

For a singular matrix, $|A| = 0$.

4.

(d) $\frac{2 \log x - 3}{x^3}$

Explanation:

$$\begin{aligned}\frac{d}{dx} \left(\frac{\log x}{x} \right) &= \frac{x \cdot \frac{1}{x} - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2} \\ \Rightarrow \frac{d}{dx} \left(\frac{1 - \log x}{x^2} \right) \\ &= \frac{x^2 \cdot (-\frac{1}{x}) - (1 - \log x) \cdot 2x}{x^4} = \frac{-x - 2x + 2x \log x}{x^4} = \frac{x(2 \log x - 3)}{x^4} = \frac{2 \log x - 3}{x^3}\end{aligned}$$

5. (a) -2

Explanation:

Given lines are $\frac{2x-2}{2k} = \frac{4-y}{3} = \frac{z+2}{-1}$ and $\frac{x-5}{1} = \frac{y}{k} = \frac{z+6}{4}$

Writing the above equation in standard form, we get

$$\Rightarrow \frac{2(x-1)}{2k} = \frac{-(y-4)}{3} = \frac{z+2}{-1}$$

$$\Leftrightarrow \frac{(x-1)}{k} = \frac{y-4}{-3} = \frac{z+2}{-1}$$

Now, the direction ratio of the first line is (k, -3, -1) and the direction ratio of second line is (1, k, 4)

Since, lines are perpendicular,

$$\therefore (k \times 1) + (-3 \times k) + (-1 \times 4) = 0$$

$$\Rightarrow k - 3k - 4 = 0$$

$$\Rightarrow -2k - 4 = 0$$

$$\therefore k = -2$$

6. (a) $y \sin x = x + C$

Explanation:

We have,

$$\frac{dy}{dx} + y \cot x = \operatorname{cosec} x$$

Comparing with $\frac{dy}{dx} + Py = Q$ of the above equation then, we get

$$\Rightarrow P = \cot x, Q = \operatorname{cosec} x$$

$$\text{I.F.} = e^{\int P dx} = e^{\int \cot x dx} = e^{\log \sin x} = \sin x$$

Multiplying on both sides by sin x

$$\sin x \frac{dy}{dx} + y \cos x = 1$$

$$\Rightarrow \frac{d}{dx} (y \sin x) = 1$$

$$\Rightarrow y \sin x = \int 1 dx$$

$$\Rightarrow y \sin x = x + C$$

7. (a) Minimum $Z = 300$ at (60, 0)

Explanation:

Objective function is $Z = 5x + 10y$ (1).

The given constraints are : $x + 2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

The corner points are obtained by drawing the lines $x+2y=120$, $x+y=60$ and $x-2y=0$. The points so obtained are (60,30), (120,0), (60,0) and (40,20)

Corner points	$Z = 5x + 10y$
D(60 ,30)	600
A(120,0)	600
B(60,0)	300.....(Min.)
C(40,20)	400

Here , $Z = 300$ is minimum at (60, 0).

8. (a) $\frac{\pi}{2}$

Explanation:

$$\text{We have, } \cos^{-1} \cos \frac{3\pi}{2}$$

We know that,

$$\cos \frac{3\pi}{2} = 0$$

$$\text{So, } \cos^{-1} \cos \frac{3\pi}{2} = \cos^{-1} 0$$

Let, $\cos^{-1} 0 = \theta$

$\Rightarrow \cos \theta = 0$

Principal value of $\cos^{-1} x$ is $[0, \pi]$

For, $\cos \theta = 0$

So, $\theta = \frac{\pi}{2}$

9.

(c) $\frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + C$

Explanation:

Put $x^3 = t$ and $3x^2 dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{9-16t^2}} = \frac{1}{4} \int \frac{dt}{\sqrt{\frac{9}{16}-t^2}} = \frac{1}{4} \cdot \int \frac{dt}{\sqrt{\left(\frac{3}{4}\right)^2-t^2}}$$

$$= \frac{1}{4} \sin^{-1} \frac{t}{(3/4)} + C = \frac{1}{4} \sin^{-1} \left(\frac{4x^3}{3} \right) + C$$

10.

(c) B + D

Explanation:

Only B + D is defined because matrices of the same order can only be added.

11.

(b) bounded

Explanation:

A feasible region of a system of linear inequalities is said to be bounded, if it can be enclosed within a circle.

12.

(b) $\frac{\pi}{4}$

Explanation:

$\frac{\pi}{4}$

Hint

$$\vec{a} = (\hat{i} + \hat{j} + \sqrt{2}\hat{k}) \Rightarrow |\vec{a}| = |\vec{a}| = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = \sqrt{4} = 2$$

Direction ratios of $\vec{a}$ are $(1, 1, \sqrt{2})$

Direction cosines of $\vec{a}$ are $(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2})$, i.e., $(\frac{1}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}})$

Direction cosines along to z-axis $(0, 0, 1)$

$$\therefore \cos \gamma = \frac{1}{\sqrt{2}} \Rightarrow \gamma = \frac{\pi}{4}$$

13.

(b) $\frac{1}{16}$

Explanation:

$\frac{1}{16}$

14.

(a) $\frac{1}{36}$

Explanation:

$\frac{1}{36}$

15.

(a) $\sin \frac{y}{x} = cx$

Explanation:

We have ,

$$x \frac{dy}{dx} = y + x \tan \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} + \tan \left(\frac{y}{x} \right) \dots (I)$$

Put $\frac{y}{x} = v$

$$y = vx$$

Differentiating on both sides,

$$\frac{dy}{dx} = v + x \frac{dv}{dx}$$

$$v + \tan v = v + x \frac{dv}{dx} \text{ ...from (i)}$$

$$\tan v = x \frac{dv}{dx}$$

$$\frac{dx}{x} = \cot v dv$$

$$\int \frac{dx}{x} = \int \cot v dv$$

$$\log |x| + \log C = \log |\sin v|$$

$$Cx = \sin v$$

$$\sin\left(\frac{y}{x}\right) = cx$$

16. (a) 1

Explanation:

$$\text{Given: } (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k}(\hat{i} \times \hat{j})$$

$$\text{i. } (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j}) = \hat{i} \cdot \hat{i} + \hat{j} \cdot (-\hat{j}) + \hat{k} \cdot \hat{k}$$

$$= 1 - \hat{j} \cdot \hat{j} + 1$$

$$= 1 - 1 + 1$$

$$= 1$$

17.

(c) is equal to 0

Explanation:

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} e^{\frac{1}{x}} = 0, \quad \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x = 0 \therefore \lim_{x \rightarrow 0} f(x) = 0$$

18.

$$(d) \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Explanation:

We know that if there are two parallel lines then their direction ratios must have a relation

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

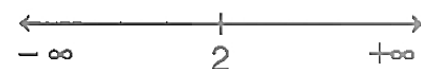
19.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

$$\text{We have, } f(x) = x^2 - 4x + 6$$

$$\text{or } f'(x) = 2x - 4 = 2(x - 2)$$



Therefore, $f'(x) = 0$ gives $x = 2$.

Now, the point $x = 2$ divides the real line into two disjoint intervals namely, $(-\infty, 2)$ and $(2, \infty)$.

In the interval $(-\infty, 2)$, $f'(x) = 2x - 4 < 0$.

Therefore, f is strictly decreasing in this interval.

Also, in the interval $(2, \infty)$, $f'(x) > 0$ and so the function f is strictly increasing in this interval.

Hence, both the statements are true but Reason is not the correct explanation of Assertion.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation:

Assertion: Here, $f: \mathbb{R} \rightarrow \mathbb{R}$ is given as

$$f(x) = x^3.$$

$$\text{Suppose } f(x) = f(y)$$

$$\text{where } x, y \in \mathbb{R}$$

$$\Rightarrow x^3 = y^3 \text{ ... (i)}$$

Now, we try to show that $x = y$

Suppose $x \neq y$, their cubes will also not be equal.

$$x^3 \neq y^3$$

However, this will be a contradiction to Eq. (i).

Therefore, $x = y$. Hence, f is injective. Hence, both Assertion and Reason are true and Reason is the correct explanation of Assertion.

Section B

21. The domain of $\sin^{-1} x$ is $[-1, 1]$. Therefore, $f(x) = \sin^{-1}(-x^2)$ is defined for all x satisfying $-1 \leq -x^2 \leq 1$

$$\Rightarrow -1 \geq x^2 \geq -1$$

$$\Rightarrow 0 \leq x^2 \leq 1$$

$$\Rightarrow x^2 \leq 1$$

$$\Rightarrow x^2 - 1 \leq 0$$

$$\Rightarrow (x - 1)(x + 1) \leq 0$$

$$\Rightarrow -1 \leq x \leq 1$$

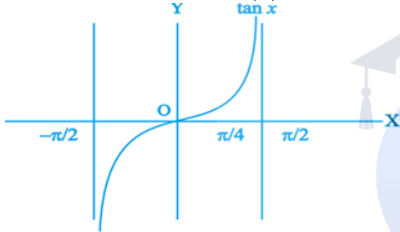
Hence, the domain of $f(x) = \sin^{-1}(-x^2)$ is $[-1, 1]$.

OR

From Fig. we note that $\tan x$ is an increasing function in the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, since $1 > \frac{\pi}{4} \Rightarrow \tan 1 > \tan \frac{\pi}{4}$. This gives $\tan 1 > 1$

$$\Rightarrow \tan 1 > 1 > \frac{\pi}{4}$$

$$\Rightarrow \tan 1 > 1 > \tan^{-1}(1)$$



22. Let x cm be the radius and y be the enclosed area of the circular wave at any time t .

Rate of increase of radius of circular wave = 5 cm/sec

$$\Rightarrow \frac{dx}{dt} \text{ is positive and } = 5 \text{ cm/sec}$$

$$\Rightarrow \frac{dx}{dt} = 5 \text{ cm/sec} \dots (i)$$

$$y = \pi x^2$$

$$\therefore \text{Rate of change of area} = \frac{dy}{dt} = \pi \frac{d}{dt} x^2$$

$$= \pi \cdot 2x \frac{dx}{dt} = 2\pi x (5) \text{ (from (i))}$$

$$= 10\pi x \text{ cm}^2 / \text{sec}$$

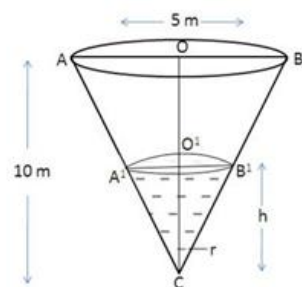
Putting $x = 8 \text{ cm}$ (given),

$$\frac{dy}{dt} = 10\pi (8) = 80\pi \text{ cm}^2 / \text{sec}$$

Since $\frac{dy}{dt}$ is positive, therefore area of circular wave is increasing at the rate of $80\pi \text{ cm}^2 / \text{sec}$.

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23.



Let α be the semi vertical angle of the cone whose height $CO = 10 \text{ m}$ and radius $OB = 5 \text{ m}$.

$$\text{Now, } \tan \alpha = \frac{OB}{CO} = \frac{5}{10}$$

$$\tan \alpha = \frac{1}{2}$$

Let V be the volume of water in the cone, then

$$v = \frac{1}{3} \pi (O'B')^2 (CO')$$

$$v = \frac{1}{3} \pi h^3 \tan^2 \alpha$$

$$v = \frac{\pi}{12} h^3$$

$$\frac{dv}{dt} = \frac{3\pi}{12} h^2 \frac{dh}{dt}$$

$$\pi = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{h^2}$$

$$\left(\frac{dh}{dt} \right)_{h=2.5} = \frac{4}{(2.5)^2}$$

$$= \frac{4}{6.25}$$

$$= 0.64 \text{ m/min.}$$

So, the water level is rising at the rate of 0.64 m/min.

OR

Given: $f(x) = x^2 - x + 1$

$$\Rightarrow f'(x) = 2x - 1$$

$f(x)$ is strictly increasing if $f'(x) > 0$

$$\Rightarrow 2x - 1 > 0$$

$$\Rightarrow x > \frac{1}{2}$$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

$f(x)$ is strictly decreasing if $f'(x) < 0$

$$\Rightarrow 2x - 1 < 0$$

$$\Rightarrow x < \frac{1}{2}$$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, $f(x)$ is neither strictly increasing nor decreasing on the interval $(-1, 1)$.

24. Let $I = \int \tan^3 x \sec^2 x \, dx$

Now let $\tan x = t$. Then, $d(\tan x) = dt \Rightarrow \sec^2 x \, dx = dt \Rightarrow dx = \frac{dt}{\sec^2 x}$

Put $\tan x = t$ and $dx = \frac{dt}{\sec^2 x}$, we get

$$I = \int \tan^3 x \sec^2 x \, dx$$

$$= \int t^3 \sec^2 x \times \frac{dt}{\sec^2 x}$$

$$= \int t^3 \, dt$$

$$= \frac{t^4}{4} + C$$

$$= \frac{\tan^4 x}{4} + C$$

25. We are given that,

Order of matrix $A = 3$

$$|A| = 4$$

We need to find the value of $|2A|$.

By the property of determinant of a matrix,

$$|KA| = K^n |A|$$

Where the order of the matrix A is n .

Similarly,

$$|2A| = 2^3 |A| \quad \dots [\because \text{Order of matrix } A = 3]$$

$$\Rightarrow |2A| = 8 |A|$$

Substituting the value of $|A|$ in the above equation,

$$\Rightarrow |2A| = 8 \times 4$$

$$\Rightarrow |2A| = 32$$

Thus, the value of $|2A|$ is 32.

Section C

26. Here the integrand $\frac{x^2+1}{x^2-5x+6}$ is not proper rational function, so we divide $x^2 + 1$ by $x^2 - 5x + 6$ and find that

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6} = 1 + \frac{5x-5}{(x-2)(x-3)}$$

$$\text{Let } \frac{5x-5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3}$$

$$\text{So that } 5x - 5 = A(x - 3) + B(x - 2)$$

Equating the coefficients of x and constant terms on both sides,

we get $A + B = 5$ and $3A + 2B = 5$.

Solving these equations, we get $A = -5$ and $B = 10$

$$\text{Thus, } \frac{x^2+1}{x^2-5x+6} = 1 - \frac{5}{x-2} + \frac{10}{x-3}$$

$$\begin{aligned} \text{Therefore, } \int \frac{x^2+1}{x^2-5x+6} dx &= \int dx - 5 \int \frac{1}{x-2} dx + 10 \int \frac{dx}{x-3} \\ &= x - 5 \log |x-2| + 10 \log |x-3| + C. \end{aligned}$$

27. A white ball can be drawn in three mutually exclusive ways:

- By transferring two black balls from first to second urn, then drawing a white ball
- By transferring two white balls from first to second urn, then drawing a white ball
- By transferring a white and a black ball from first to second urn, then drawing a white ball

Consider the following events:

E_1 = Two black balls are transferred from first to second bag

E_2 = Two white balls are transferred from first to second bag

E_3 = A white and a black ball is transferred from first to second bag

A = A white ball is drawn

Therefore, we have,

$$P(E_1) = \frac{{}^3C_2}{{}^{13}C_2} = \frac{3}{78}$$

$$P(E_2) = \frac{{}^{10}C_2}{{}^{13}C_2} = \frac{45}{78}$$

$$P(E_3) = \frac{{}^{10}C_1 \times {}^3C_1}{{}^{13}C_2} = \frac{30}{78}$$

Now,

$$P\left(\frac{A}{E_1}\right) = \frac{3}{10}$$

$$P\left(\frac{A}{E_2}\right) = \frac{5}{10}$$

$$P\left(\frac{A}{E_3}\right) = \frac{4}{10}$$

Using the law of total probability, we get

$$\text{Required probability} = P(A) = P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right) + P(E_3)P\left(\frac{A}{E_3}\right)$$

$$\begin{aligned} &= \frac{3}{78} \times \frac{3}{10} + \frac{45}{78} \times \frac{5}{10} + \frac{30}{78} \times \frac{4}{10} \\ &= \frac{9}{780} + \frac{225}{780} + \frac{120}{780} \\ &= \frac{354}{780} = \frac{59}{130} \end{aligned}$$

28. According to the question, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{\sin x + \cos x} dx$... (i)

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx$$

$$\left[\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right]$$

$$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{\cos x + \sin x} dx \text{ ... (ii)}$$

Adding Equations (i) and (ii),

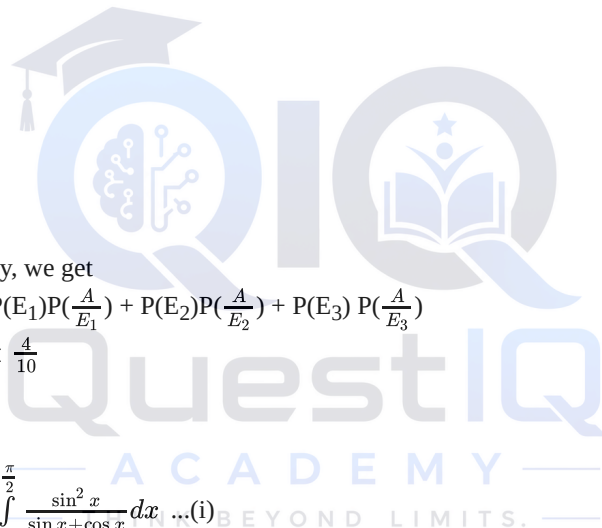
$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x + \cos^2 x}{\sin x + \cos x} dx$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx$$

$$[\because \sin^2 x + \cos^2 x = 1]$$

$$\Rightarrow 2I = \int_0^{\frac{\pi}{2}} \frac{1}{\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} + \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}} dx$$

$$\left[\because \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \text{ and } \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right]$$



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$$= \int_0^{\frac{\pi}{2}} \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$\text{Put } \tan \frac{x}{2} = t \Rightarrow \sec^2 \frac{x}{2} \cdot \frac{1}{2} dx = dt \Rightarrow \sec^2 \frac{x}{2} dx = 2dt$$

Lower limit when $x = 0$, then $t = \tan 0 = 0$

Upper limit when $x = \frac{\pi}{2}$, then $t = \tan \frac{\pi}{4} = 1$.

$$\therefore 2I = \int_0^1 \frac{2dt}{2t+1-t^2} = 2 \int_0^1 \frac{dt}{-[t^2-2t-1]}$$

$$= 2 \int_0^1 \frac{dt}{-[(t-1)^2-1-1]} = 2 \int_0^1 \frac{dt}{(\sqrt{2})^2-(t-1)^2}$$

$$= \left[\frac{2}{2\sqrt{2}} \log \left| \frac{\sqrt{2}+t-1}{\sqrt{2}-t-1} \right| \right]_0^1$$

$$\left[\because \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C \right]$$

$$= \frac{1}{\sqrt{2}} \left[\log \frac{\sqrt{2}+1-1}{\sqrt{2}-1-1} - \log \frac{\sqrt{2}+0-1}{\sqrt{2}-0-1} \right]$$

$$= \frac{1}{\sqrt{2}} \left[\log 1 - \log \frac{\sqrt{2}-1}{\sqrt{2}+1} \right]$$

$$= -\frac{1}{\sqrt{2}} \log \left[\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} \right] [\because \log 1 = 0]$$

$$= -\frac{1}{\sqrt{2}} \log \frac{2-1}{(\sqrt{2}+1)^2} [\because (a-b)(a+b) = a^2 - b^2]$$

$$= -\frac{1}{\sqrt{2}} \log \frac{1}{(\sqrt{2}+1)^2}$$

$$\Rightarrow 2I = \frac{2}{\sqrt{2}} \log(\sqrt{2}+1) \Rightarrow I = \frac{1}{\sqrt{2}} \log(\sqrt{2}+1)$$

OR

$$\text{Let } I = \int \frac{(x-1)^2}{x^2+2x+2} dx$$

$$= \int \left(\frac{x^2-2x+1}{x^2+2x+2} \right) dx$$

Therefore by long division we have,

$$\begin{array}{r} x^2 + 2x + 2 \overline{) x^2 - 2x + 1} \\ \underline{-(x^2 + 2x + 2)} \\ -4x - 1 \end{array}$$

Therefore,

$$\frac{x^2-2x+1}{x^2+2x+2} = 1 - \frac{(4x+1)}{x^2+2x+2} \dots(i)$$

$$\text{Let } 4x+1 = A \frac{d}{dx}(x^2+2x+2) + B$$

$$4x+1 = A(2x+2) + B$$

$$4x+1 = (2A)x + 2A + B$$

Equating Coefficients of like terms

$$2A = 4$$

$$A = 2$$

$$2A + B = 1$$

$$2 \times 2 + B = 1$$

$$B = -3$$

$$\int \left(\frac{x^2-2x+1}{x^2+2x+2} \right) dx$$

$$= \int dx - 2 \int \frac{(2x+2)}{x^2+2x+2} dx + 3 \int \frac{dx}{x^2+2x+2}$$

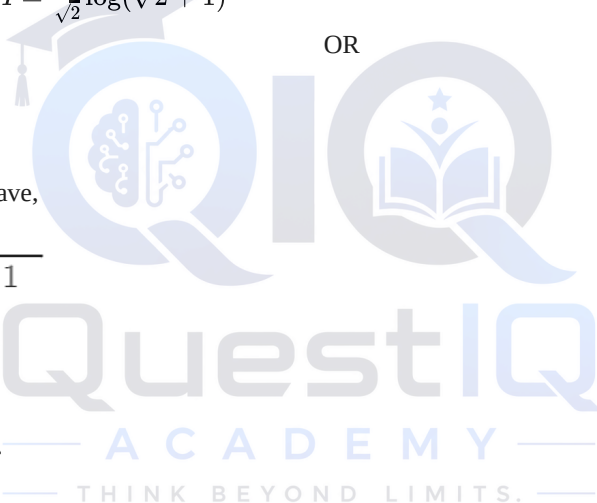
$$= \int dx - 2 \int \frac{(2x+2)}{x^2+2x+2} dx + 3 \int \frac{dx}{(x+1)^2+1^2}$$

$$= x - 2 \log |x+2x+2| + \frac{3}{1} \tan^{-1} \left(\frac{x+1}{1} \right) + C$$

$$= x - 2 \log |x+2x+2| + 3 \tan^{-1} (x+1) + C$$

$$29. \text{ It is given that } (x-y)(dx+dy) = dx-dy$$

$$\Rightarrow (x-y+1)dy = (1-x+y)dx$$



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$$\Rightarrow \frac{dy}{dx} = \frac{1-x+y}{x-y+1} \dots\dots(i)$$

Let $x - y = t$

$$\Rightarrow \frac{d}{dx}(x - y) = \frac{dt}{dx}$$

$$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$$

Now, let us substitute the value of $x-y$ and $\frac{dy}{dx}$ in equation (i), we get,

$$1 - \frac{dt}{dx} = \frac{1-t}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = 1 - \left(\frac{1-t}{1+t}\right)$$

$$\Rightarrow \frac{dt}{dx} = \frac{(1+t)-(1-t)}{1+t}$$

$$\Rightarrow \frac{dt}{dx} = \frac{2t}{1+t}$$

$$\Rightarrow \left(\frac{1+t}{t}\right) dt = 2dx$$

$$\Rightarrow \left(1 + \frac{1}{t}\right) dt = 2dx \dots\dots(ii)$$

On integrating both side, we get,

$$t + \log|t| = 2x + C$$

$$\Rightarrow (x - y) + \log|x - y| = 2x + C$$

$$\Rightarrow \log|x - y| = x + y + C \dots\dots(iii)$$

Now, $y = -1$ at $x = 0$

Then, equation (iii), we get,

$$\log 1 = 0 - 1 + C$$

$$\Rightarrow C = 1$$

Substituting $C = 1$ in equation (iii), we get,

$$\log|x - y| = x + y + 1$$

Therefore, a particular solution of the given differential equation is $\log|x - y| = x + y + 1$.

OR

We have to solve,

$$x \log|x| \frac{dy}{dx} + y = \frac{2}{x} \log|x|$$

On dividing both sides by $x \log|x|$, we get

$$\frac{dy}{dx} + \frac{y}{x \log|x|} = \frac{2}{x^2} \frac{\log|x|}{\log|x|} = \frac{2}{x^2}$$

which is a linear differential equation of first order, which is of the form of $\frac{dy}{dx} + Py = Q$,

$$\text{Here, } P = \frac{1}{x \log|x|} \text{ and } Q = \frac{2}{x^2}$$

We know that,

$$\text{IF} = e^{\int P dx} = e^{\int \frac{1}{x \log|x|} dx}$$

$$\text{put } \log|x| = t \Rightarrow \frac{1}{x} dx = dt$$

$$\therefore \text{IF} = \int \frac{1}{t} dt = \log|t| = \log|\log|x||$$

$$\Rightarrow \text{IF} = \log|x| \quad [\because e^{\log x} = x]$$

Now, solution of above equation is given by

$$y \times \text{IF} = \int (Q \times \text{IF}) dx + C$$

$$y \log|x| = \int \frac{2}{x^2} \log|x| dx + C$$

$$\Rightarrow y \log|x| = 2 \left[\log|x| \int \frac{1}{x^2} dx - \int \left(\frac{d}{dx}(\log|x|) \cdot \int \frac{1}{x^2} dx \right) dx \right] + C \text{ [using integration by parts]}$$

$$\Rightarrow y \log|x| = 2 \left[\log|x| \cdot \left(-\frac{1}{x}\right) - \int \frac{1}{x} \cdot \left(-\frac{1}{x}\right) dx + C \right]$$

$$\Rightarrow y \log|x| = 2 \left[-\frac{1}{x} \log|x| + \int \frac{1}{x^2} dx \right] + C$$

$$\therefore y \log|x| = -\frac{2}{x} \log|x| - \frac{2}{x} + C$$

30. According to the question,

$$\vec{a} = 3\hat{i} - \hat{j} \text{ and}$$

$$\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$$

$$\text{Let } \vec{b}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \text{ and}$$

$$\vec{b}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\vec{b}_1 + \vec{b}_2 = \vec{b}, \vec{b}_1 \parallel \vec{a} \text{ and } \vec{b}_2 \perp \vec{a}.$$

$$\text{Consider, } \vec{b}_1 + \vec{b}_2 = \vec{b}$$

$$\Rightarrow (x_1 + x_2)\hat{i} + (y_1 + y_2)\hat{j} + (z_1 + z_2)\hat{k} = 2\hat{i} + \hat{j} - 3\hat{k}$$

On comparing the coefficient of $\hat{i}$, $\hat{j}$ and $\hat{k}$ both sides; we get

$$\Rightarrow x_1 + x_2 = 2 \dots(i)$$

$$y_1 + y_2 = 1 \dots(ii)$$

$$\text{and } z_1 + z_2 = -3 \dots(iii)$$

$$\text{Now, consider } \vec{b}_1 \parallel \vec{a}$$

$$\Rightarrow \frac{x_1}{3} = \frac{y_1}{-1} = \frac{z_1}{0} = \lambda(\text{say})$$

$$\Rightarrow x_1 = 3\lambda, y_1 = -\lambda \text{ and } z_1 = 0 \dots(iv)$$

On substituting the values of x,y and z, form Eq. (iv) to Eq. (i), (ii) and (iii), respectively, we get

$$x_2 = 2 - 3\lambda, y_2 = 1 + \lambda \text{ and } z_2 = -3 \dots(v)$$

$$\text{Since, } \vec{b}_2 \perp \vec{a}, \text{ therefore } \vec{b}_2 \cdot \vec{a} = 0$$

$$\Rightarrow 3x_2 - y_2 = 0$$

$$\Rightarrow 3(2 - 3\lambda) - (1 + \lambda) = 0$$

$$\Rightarrow 6 - 9\lambda - 1 - \lambda = 0$$

$$\Rightarrow 5 - 10\lambda = 0 \Rightarrow \lambda = \frac{1}{2}$$

On substituting $\lambda = \frac{1}{2}$ in Eqs. (iv) and Eqs. (iv) and (v), we get

$$x_1 = \frac{3}{2}, y_1 = -\frac{1}{2}, z_1 = 0$$

$$\text{and } x_2 = \frac{1}{2}, y_2 = \frac{3}{2} \text{ and } z_2 = -3$$

$$\text{Hence, } \vec{b} = \vec{b}_1 + \vec{b}_2 = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}\right) = 2\hat{i} + \hat{j} - 3\hat{k}$$

OR

Given ,

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0 \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0 \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} \perp (\vec{b} - \vec{c}) [\because \vec{a} \neq \vec{0}]$$

$$\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \perp (\vec{b} - \vec{c}) \dots(i)$$

Again given,

$$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0} \text{ and } \vec{a} \neq \vec{0}$$

$$\Rightarrow \vec{b} - \vec{c} = \vec{0} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) [\because \vec{a} \neq \vec{0}]$$

$$\Rightarrow \vec{b} = \vec{c} \text{ or } \vec{a} \parallel (\vec{b} - \vec{c}) \dots(ii)$$

From (i) and (ii), it follows that $\vec{b} = \vec{c}$, because $\vec{a}$ cannot be both parallel and perpendicular to vectors $(\vec{b} - \vec{c})$

31. We have, $x = \sin t$ and $y = \sin pt$,

$$\therefore \frac{dx}{dt} = \cos t \text{ and } \frac{dy}{dt} = \cos pt \cdot p$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{p \cdot \cos pt}{\cos t} \dots(i)$$

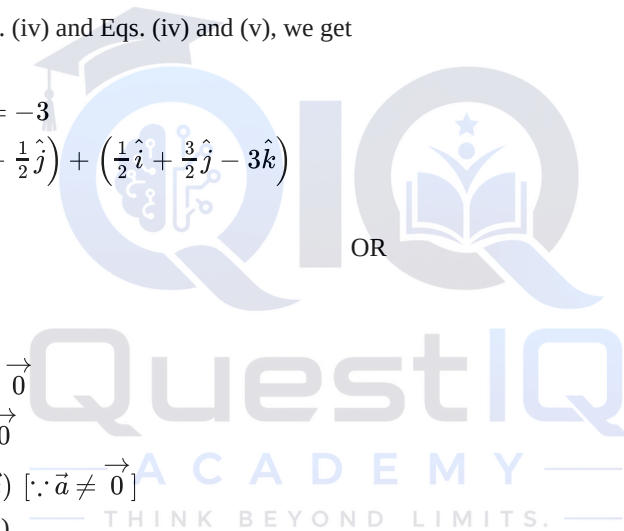
Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = \frac{\cos t \cdot \frac{d}{dt}(p \cdot \cos pt) \frac{dt}{dx} - p \cos pt \cdot \frac{d}{dt} \cos t \cdot \frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{[\cos t \cdot p \cdot (-\sin pt) \cdot p - p \cos pt \cdot (-\sin t)] \frac{dt}{dx}}{\cos^2 t}$$

$$= \frac{[-p^2 \sin pt \cdot \cos t + p \sin t \cdot \cos pt] \cdot \frac{1}{\cos t}}{\cos^2 t}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t}{\cos^3 t} \dots(ii)$$



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Since, we have to prove

$$(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} + p^2 y = 0$$

$$\therefore LHS = (1 - \sin^2 t) \frac{[-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t]}{\cos^3 t}$$

$$- \sin t \cdot \frac{p \cos pt}{\cos t} + p^2 \sin pt$$

$$= \frac{1}{\cos^3 t} \left[(1 - \sin^2 t) (-p^2 \sin pt \cdot \cos t + p \cos pt \cdot \sin t) \right]$$

$$= \frac{1}{\cos^3 t} \left[-p^2 \sin pt \cdot \cos^3 t + p \cos pt \cdot \sin t \cdot \cos^2 t \right] [\because 1 - \sin^2 t = \cos^2 t]$$

$$= \frac{1}{\cos^3 t} \cdot 0$$

= 0 Hence proved.

Section D

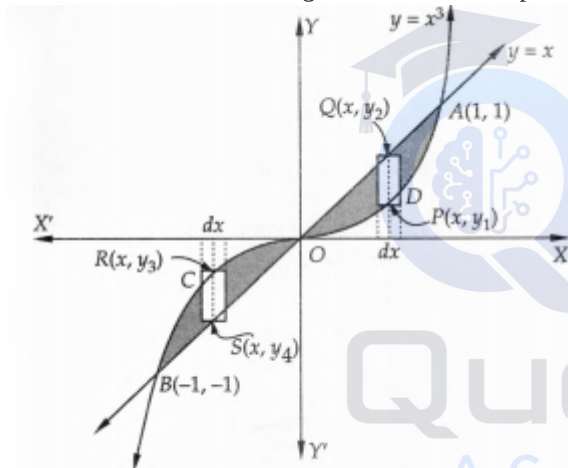
32. The given curves are ,

$$y = x \dots (i)$$

$$\text{and } y = x^3 \dots (ii)$$

The sketch of the curve $y = x^3$ is shown in Fig. Clearly, $y = x$ is a line passing through the origin and making an angle of 45° with x -axis. The shaded portion shown in Fig. is the region bounded by the curves $y = x$ and $y = x^3$. Solving $y = x$ and $y = x^3$ simultaneously, we find that the two curves intersect at $O(0, 0)$, $A(1, 1)$ and $B(-1, -1)$.

When we slice the shaded region into vertical strips, we observe that the vertical strips change their character at O .



Therefore, the required area is given by,

Required area = Area BCOB + Area ODAO

Area BCOB: Each vertical strip in this region has its lower end on $y = x$ and the upper end on $y = x^3$. Therefore, the approximating rectangle shown in this region has length = $|y_4 - y_3|$, width = dx and area = $|y_4 - y_3| dx$. Since the approximating rectangle can move from $x = -1$ to $x = 0$.

$$\begin{aligned} \therefore \text{Area BCOB} &= \int_{-1}^0 |y_4 - y_3| dx = \int_{-1}^0 -(y_4 - y_3) dx \quad [\because y_4 < y_3 \therefore y_4 - y_3 < 0] \\ &= \int_{-1}^0 -(x - x^3) dx \quad [\because R(x, y_3) \text{ and } S(x, y_4) \text{ lie on (ii) and (i) respectively } \therefore y_3 = x^3 \text{ and } y_4 = x] \\ &= \int_{-1}^0 (x^3 - x) dx = \left[\frac{x^4}{4} - \frac{x^2}{2} \right]_{-1}^0 = 0 - \left(\frac{1}{4} - \frac{1}{2} \right) = \frac{1}{4} \text{ sq. units} \end{aligned}$$

Area ODAO: Each vertical strip in this region has its two ends on (ii) and (i) respectively. So, the approximating rectangle shown in this region has length = $|y_2 - y_1|$, width = dx and therefore, we have,

$$\begin{aligned} \text{Area ODAO} &= \int_0^1 |y_2 - y_1| dx = \int_0^1 (y_2 - y_1) dx \quad [\because y_2 > y_1 \therefore y_2 - y_1 > 0] \\ &= \int_0^1 (x - x^3) dx \quad [\because P(x, y_1) \text{ and } Q(x, y_2) \text{ lie on (ii) and (i) respectively } \therefore y_1 = x^3 \text{ and } y_2 = x] \\ &= \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4} \text{ sq. units} \end{aligned}$$

$$\therefore \text{Required area} = \text{Area BCOB} + \text{Area ODAO} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ sq. units}$$

33. $R = \{(a, b) = |a \cdot b| \text{ is divisible by } 2\}$

where $a, b \in A = \{1, 2, 3, 4, 5\}$

reflexivity

For any $a \in A, |a - a| = 0$ Which is divisible by 2.

$\therefore (a, a) \in r$ for all $a \in A$

So, R is Reflexive

Symmetric :

Let $(a, b) \in R$ for all $a, b \in R$

$|a-b|$ is divisible by 2

$|b-a|$ is divisible by 2

$(a, b) \in R \Rightarrow (b, a) \in R$

So, R is symmetric.

Transitive :

Let $(a, b) \in R$ and $(b, c) \in R$ then

$(a, b) \in R$ and $(b, c) \in R$

$|a-b|$ is divisible by 2

$|b-c|$ is divisible by 2

Two cases :

Case 1:

When b is even

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

$\therefore (a, c) \in R$

Case 2:

When b is odd

$(a, b) \in R$ and $(b, c) \in R$

$|a-c|$ is divisible by 2

$|b-c|$ is divisible by 2

$|a-c|$ is divisible by 2

Thus, $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$

So R is transitive.

Hence, R is an equivalence relation

OR

Given that $A = [-1, 1]$

i. $f(x) = \frac{x}{2}$

Let $f(x_1) = f(x_2)$

$$\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$$

So, $f(x)$ is one-one.

Now, let $y = \frac{x}{2}$

$$\Rightarrow x = 2y \notin A, \forall y \in A$$

As for $y = 1 \in A$, $x = 2 \notin A$

So, $f(x)$ is not onto.

Also, $f(x)$ is not bijective as it is not onto.

ii. $g(x) = |x|$

Let $g(x_1) = g(x_2)$

$$\Rightarrow |x_1| = |x_2| \Rightarrow x_1 = \pm x_2$$

So, $g(x)$ is not one-one.

Now, $x = \pm y \notin A$ for all $y \in R$

So, $g(x)$ is not onto, also, $g(x)$ is not bijective.

iii. $h(x) = x|x|$

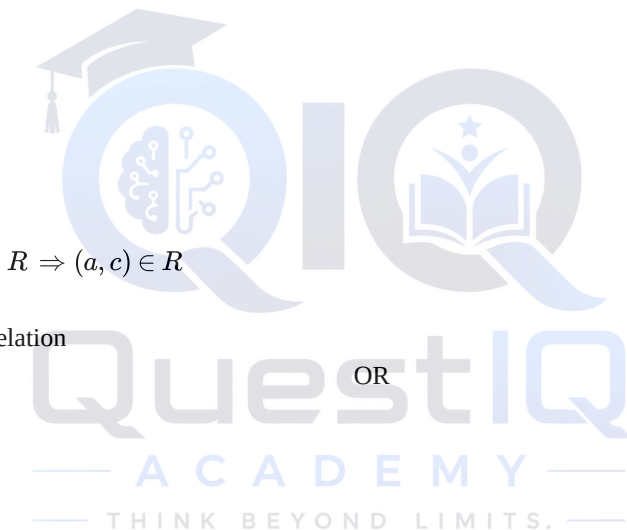
$$\Rightarrow x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$$

So, $h(x)$ is one-one

Now, let $y = x|x|$

$$\Rightarrow y = x^2 \in A, \forall x \in A$$

So, $h(x)$ is onto also, $h(x)$ is a bijective.



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iv. $k(x) = x^2$

Let $k(x_1) = k(x_2)$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

Thus, $k(x)$ is not one-one.

Now, let $y = x^2$

$\Rightarrow x\sqrt{y} \notin A, \forall y \in A \quad x = \sqrt{y} \notin A, \forall y \in A$

As for $y = -1, x = \sqrt{-1} \notin A$

Hence, $k(x)$ is neither one-one nor onto.

34. On adding

$$5x + 5y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

$$5(x + y) = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$$

$$(x + y) = \frac{1}{5} \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix} \text{-----(i)}$$

On subtracting

$$x - y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{-----(ii)}$$

$$x - y = \begin{bmatrix} 0 & -5 \\ -5 & 5 \end{bmatrix}$$

Adding (i) and (ii) gives,

$$2x = \begin{bmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{bmatrix}$$

$$x = \begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix}$$

$$x + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix}$$

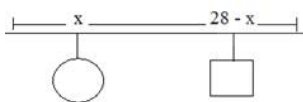
$$\begin{bmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{bmatrix} + y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{5} \end{bmatrix}$$

$$y = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{3}{5} & 1 \end{bmatrix} + \begin{bmatrix} \frac{-2}{5} & \frac{12}{5} \\ \frac{11}{5} & -3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{5} - \frac{2}{5} & \frac{1}{5} + \frac{12}{5} \\ \frac{3}{5} + \frac{11}{5} & 1 - 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$$

35. Let 1st length = x

2nd length = 28 - x



Now circumference of circle is $2\pi r$

$\therefore 2\pi r = x$

$\Rightarrow r = \frac{x}{2\pi}$

Now perimeter of rectangle = 4a

$\therefore 4a = 28 - x$

$\Rightarrow a = 7 - \frac{x}{4}$

ATQ

A = area of circle + area of square

$\pi \left(\frac{x}{2\pi} \right)^2 + \left(7 - \frac{x}{4} \right)^2$

Now, $A = \pi \cdot \frac{x^2}{4\pi^2} + \left(7 - \frac{x}{4} \right)^2$

So, $\frac{dA}{dx} = \frac{2x}{4\pi} + 2 \left(7 - \frac{x}{4} \right) \left(-\frac{1}{4} \right)$



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$$\frac{dA}{dx} = 0$$

$$\Rightarrow \frac{2x}{4\pi} + 2 \left(7 - \frac{x}{4} \right) \left(-\frac{1}{4} \right) = 0$$

$$\Rightarrow \frac{1}{2} \left(7 - \frac{x}{4} \right) = \frac{x}{2\pi}$$

$$\Rightarrow 7 - \frac{x}{4} = \frac{x}{\pi}$$

$$\Rightarrow 7 = \frac{x}{\pi} + \frac{x}{4}$$

$$\Rightarrow 7 = x \left(\frac{4+\pi}{4\pi} \right)$$

$$\Rightarrow \frac{28\pi}{4+\pi} = x$$

$$\text{Now, } \frac{d^2y}{dx^2} = \frac{1}{2\pi} - \frac{1}{2} \left(\frac{-1}{4} \right)$$

$$= \frac{1}{2\pi} + \frac{1}{8}$$

positive, hence minimum

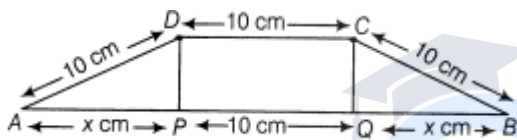
$$\text{Therefore, 1}^{\text{st}} \text{ length} = \frac{28\pi}{4+\pi}$$

$$2^{\text{nd}} \text{ length} = \frac{28}{1} - \frac{28\pi}{4+\pi}$$

$$= 28 \left[\frac{4+\pi-\pi}{4+\pi} \right]$$

$$= \frac{112}{4+\pi}$$

OR



Let ABCD be the given trapezium in which $AD = BC = CD = 10$ cm.

Draw a perpendicular DP and CQ on AB. Let $AP = x$ cm

In $\triangle APD$ & $\triangle BQC$,

$$\angle APD = \angle BQC \text{ [each } = 90^\circ]$$

$$AD = BC \text{ [Both 10 cm]}$$

$$DP = CQ \text{ [Perpendicular between parallel lines are equal in length]}$$

$$\therefore \triangle APD \cong \triangle BQC \text{ [RHS Congruency]}$$

$$\therefore QB = AP \text{ [CPCT]}$$

$$\Rightarrow QB = x \text{ cm}$$

$$DP = \sqrt{10^2 - x^2} \text{ [by Pythagoras theorem]}$$

Now, area of trapezium,

$$A = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$$

$$= \frac{1}{2} \times (2x + 10 + 10) \times \sqrt{100 - x^2}$$

$$= (x + 10)\sqrt{100 - x^2} \dots (i)$$

We need to find the area of trapezium when it is maximum i.e. we need to maximize area.

On differentiating both sides of eq(i) w.r.t.x, we get

$$\frac{dA}{dx} = (x + 10) \frac{(-2x)}{2\sqrt{100-x^2}} + \sqrt{100 - x^2}$$

$$= \frac{-x^2 - 10x + 100 - x^2}{\sqrt{100 - x^2}}$$

$$= \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}} \dots (ii)$$

$$\text{For maxima or minima, put } \frac{dA}{dx} = 0$$

$$\Rightarrow \frac{-2x^2 - 10x + 100}{\sqrt{100 - x^2}} = 0$$

$$= -2(x^2 + 5x - 50) = 0$$

$$= -2(x+10)(x-5) = 0$$

$$x = 5 \text{ or } -10$$

Since, x represents distance, so it cannot be negative.

Therefore, we take $x = 5$.

On differentiating both sides of eq.(ii) w.r.t.x, we get

$$\begin{aligned}\frac{d^2 A}{dx^2} &= \frac{\sqrt{100-x^2} \cdot \frac{d}{dx}(-2x^2-10x+100) - (-2x^2-10x+100) \frac{d}{dx}(\sqrt{100-x^2})}{(\sqrt{100-x^2})^2} \quad [\text{by using the quotient rule of derivative}] \\ &= \frac{\sqrt{100-x^2} \cdot (-4x-10) - (-2x^2-10x+100) \left(\frac{-2x}{2\sqrt{100-x^2}}\right)}{(\sqrt{100-x^2})^2} \\ &= \frac{\sqrt{100-x^2} \cdot (-4x-10) + \frac{x(-2x^2-10x+100)}{\sqrt{100-x^2}}}{100-x^2} \\ &= \frac{(100-x^2) \cdot (-4x-10) + x(-2x^2-10x+100)}{(100-x^2)^{\frac{3}{2}}} \\ &= \frac{-400x+4x^3-1000+10x^2-2x^3-10x^2+100x}{(100-x^2)^{\frac{3}{2}}}\end{aligned}$$

$$\therefore \frac{d^2 A}{dx^2} = \frac{2x^3-300x-1000}{(100-x^2)^{3/2}}$$

When $x = 5$,

$$\begin{aligned}\frac{d^2 A}{dx^2} &= \frac{2(5)^3-300(5)-1000}{[100-(5)^2]^{3/2}} \\ &= \frac{250-1500-1000}{(100-25)^{3/2}} = \frac{-2250}{75\sqrt{75}} < 0\end{aligned}$$

$\therefore$ It is maximum when $x = 5$

Thus, area of trapezium is maximum at $x = 5$ and maximum area is

$$\begin{aligned}A_{\max} &= (5+10)\sqrt{100-(5)^2} \quad [\text{put } x = 5 \text{ in Eq. (i)}] \\ &= 15\sqrt{100-25} = 15\sqrt{75} = 75\sqrt{3} \text{ cm}^2\end{aligned}$$

Section E

36. i. $P\left(\frac{L}{C}\right) = \frac{17}{100}$

ii. $P\left(\frac{L}{A}\right) = 1 - P\left(\frac{L}{B}\right) = 1 - \frac{24}{100} = \frac{76}{100}$ or $\frac{19}{25}$

iii. $P\left(\frac{A}{L}\right) = \frac{\frac{1}{4} \times \frac{24}{100}}{\frac{1}{4} \times \frac{24}{100} + \frac{1}{4} \times \frac{22}{100} + \frac{1}{4} \times \frac{17}{100} + \frac{1}{4} \times \frac{9}{100}} = \frac{24}{72} = \frac{1}{3}$

Probability that a randomly selected child is left-handed given that exactly one of the parents is left-handed.

$$= P\left(\frac{L}{B \cup C}\right) = \frac{22}{100} + \frac{17}{100} = \frac{39}{100}$$

37. i. The line along which motorcycle A is running, $\vec{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k})$, which can be rewritten as

$$\begin{aligned}(x\hat{i} + y\hat{j} + z\hat{k}) &= \lambda\hat{i} + 2\lambda\hat{j} - \lambda\hat{k} \\ \Rightarrow x &= \lambda, y = 2\lambda, z = -\lambda \Rightarrow \frac{x}{1} = \lambda, \frac{y}{2} = \lambda, \frac{z}{-1} = \lambda \\ \text{Thus, the required cartesian equation is } \frac{x}{1} &= \frac{y}{2} = \frac{z}{-1}\end{aligned}$$

ii. Clearly, D.R.'s of the required line are $\langle 1, 2, -1 \rangle$

$\therefore$ D.C.'s are

$$\left(\frac{1}{\sqrt{1^2+2^2+(-1)^2}}, \frac{2}{\sqrt{1^2+2^2+(-1)^2}}, \frac{-1}{\sqrt{1^2+2^2+(-1)^2}}\right)$$

i.e., $\left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$

iii. The line along which motorcycle B is running, is $\vec{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$, which is parallel to the vector $2\hat{i} + \hat{j} + \hat{k}$.

$\therefore$ D.R.'s of the required line are $\langle 2, 1, 1 \rangle$.

OR

Here, $\vec{a}_1 = 0\hat{i} + 0\hat{j} + 0\hat{k}$, $\vec{a}_2 = 3\hat{i} + 3\hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + \hat{k}$

$$\therefore \vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j}$$

$$\text{and } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & 1 & 1 \end{vmatrix} = 3\hat{i} - 3\hat{j} - 3\hat{k}$$

$$\text{Now, } (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{i} + 3\hat{j}) \cdot (3\hat{i} - 3\hat{j} - 3\hat{k})$$

$$= 9 - 9 = 0$$

Hence, shortest distance between the given lines is 0.

38. i.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Minimum value of Z is -48 which occurs at (0, 8).

ii.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum value of Z is 20, which occurs at (5, 0).

iii.

Corner Points	Value of $Z = 4x - 6y$
(0, 3)	$4 \times 0 - 6 \times 3 = -18$
(5, 0)	$4 \times 5 - 6 \times 0 = 20$
(6, 8)	$4 \times 6 - 6 \times 8 = -24$
(0, 8)	$4 \times 0 - 6 \times 8 = -48$

Maximum of Z - Minimum of $Z = 20 - (-48) = 20 + 48 = 68$

OR

The corner points of the feasible region are O(0, 0), A(3, 0), B(3, 2), C(2, 3), D(0, 3).



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