Section A

1.

(d) 1

Explanation:

1

2.

(d) 36

Explanation:

36

|adj A|= $|A|^{n-1}$

$=(6)^{3-1}$

= 36

3.

(c) 0

Explanation:

0

4.

(d) A

Explanation:

$A^2 = I$

$A^{-1}A^2 = A^{-1}I$

$A = A^{-1}$

5.

(c) $60^0 or 120^0$

Explanation:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\alpha = 45, \beta = 60$

put the values in above equation

$(1/\sqrt{2})^2 + (1/2)^2 + \cos^2\gamma = 1$

$\cos\gamma = \pm 1/2$

6.

(c) $\frac{1}{\sqrt{1-y^2}}$

Explanation:

It is given that $\left(1-y^2\right)\frac{dx}{dy} + yx = ay$

$\Rightarrow \frac{dx}{dy} + \frac{yx}{1-y^2} = \frac{ay}{1-y^2}$

This is equation in the form of $\frac{dx}{dy} + px = Q$ (where $p = \frac{y}{1-y^2}$ and Q = $\frac{ay}{1-y^2}$)

Now, I.F. = $e^{\int pdy} = e^{\int \frac{y}{1-y^2}dy} = e^{-\frac{1}{2}\log(1-y^2)} = e^{\log\left[\frac{1}{\sqrt{(1-y^2)}}\right]}$
= $\frac{1}{\sqrt{(1-y^2)}}$

7.

(c) infinite

Explanation:

In a LPP, if the objective f^n Z = ax + by has the maximum value on two corner point of the feasible region then every point on the line segment joining these two points gives the same maximum value.

hence, Z_{max} occurs at infinite no of times.

8.

(b) $\hat{i} \cdot \hat{k} = 0$

Explanation:

Dot product of the mutually perpendicular vector is always zero.

here $\hat{i}$ and $\hat{k}$ are perpendicular to each other

hence, $\hat{i} \cdot \hat{k} = 0$

9. **(a)** $\frac{2^{x+2}}{\log 2} + C$

Explanation:

$\frac{2^{x+2}}{\log 2} + C$

10.

(c) 8

Explanation:

8

11.

(d) bounded in first quadrant

Explanation:

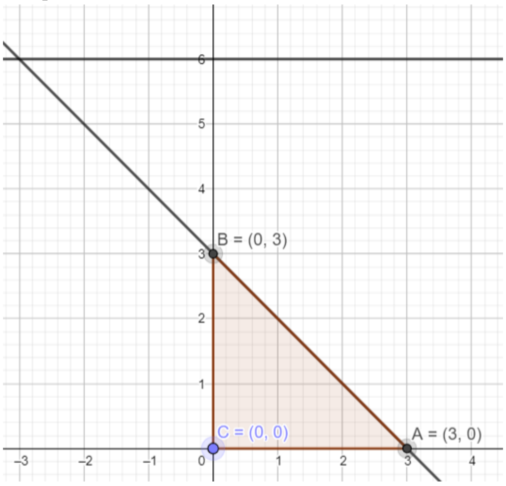
Converting the given inequations into equations, we obtain

y = 6, x + y = 3, x = 0 and y = 0, y = 6 is the line passing through (0, 6) and parallel to the X axis.The region below the line y = 6 will satisfy the given inequation.

The line x + y = 3 meets the coordinate axis at A(3, 0) and B(0, 3). Join these points to obtain the line x + y = 3 Clearly, (0, 0) satisfies the inequation x + y $\leq$3 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

Since every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.



12.

(c) $11\hat{i} + 9\hat{j} - 2\hat{k}$

Explanation:

$11\hat{i} + 9\hat{j} - 2\hat{k}$

13.

(b) d^{n-1}

Explanation:

$|A| = d$

$|adjA| = |A|^{n-1}$

$|adjA| = d^{n-1}$

14. **(a)** $\frac{1}{52}$

Explanation:

Let, E_1, E_2 and E_3 are events of selection of a scooter driver, car driver and truck driver respectively.

$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}$, $P(E_2) = \frac{4000}{12000} = \frac{1}{3}$,$P(E_3) = \frac{6000}{12000} = \frac{1}{2}$.

Let A = event that the insured person meet with the accident.

$\therefore P(A|E_1) = 0.01$, $P(A|E_2) = 0.03$, $P(A|E_3) = 0.15$

$\Rightarrow P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$

$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15} = \frac{1}{52}$

15. **(a)** 4

Explanation:

4, because the no. of arbitrary constants is equal to order of the differential equation.

16.

(d) $2\hat{i} + 3\hat{k}$

Explanation:

$2\hat{i} + 3\hat{k}$

17.

(c) $\frac{\sqrt{e^x-1}}{2}$

Explanation:

Given, $x = \log(1 + t^2)$ and $y = t - \tan^{-1} t$

On differentiating both sides w.r.t.x, we get

$\frac{dx}{dt} = \frac{1}{1+t^2}(2t)$ and $\frac{dy}{dt}$ = 1 - $\frac{1}{1+t^2}$ = $\frac{t^2}{1+t^2}$

$\therefore \frac{dy}{dx} = \frac{\frac{t^2}{(1+t^2)}}{\frac{2t}{(1+t^2)}} = \frac{t}{2}$ ...(i)

Also, x = log (1 + t^2)

$\Rightarrow$ $t^2 = e^x$-1 ...(ii)

From Eqs. (i) and (ii), we get

$\frac{dy}{dx} = \frac{\sqrt{e^x-1}}{2}$

18.

(b) $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

Explanation:

Rewrite the given line as

$r\frac{2\left(x-\frac{1}{2}\right)}{\sqrt{3}} = \frac{y+2}{2} = \frac{z-3}{3}$

or $\frac{x-\frac{1}{2}}{\sqrt{3}} = \frac{y+2}{4} = \frac{z-3}{6}$

$\therefore$ DR's of line are $\sqrt{3}$, 4 and 6

Therefore, direction cosines are:

$\frac{\sqrt{3}}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{4}{\sqrt{(\sqrt{3})^2+4^2+6^2}}, \frac{6}{\sqrt{(\sqrt{3})^2+4^2+6^2}}$ or $\frac{\sqrt{3}}{\sqrt{55}}, \frac{4}{\sqrt{55}}, \frac{6}{\sqrt{55}}$

19.

(c) A is true but R is false.

Explanation:

Let S(x) be the selling price of x items and let C(x) be the cost price of x items.

Then, we have

S(x) = (5 - $\frac{x}{100}$)x = 5x - $\frac{x^2}{100}$

and C(x) = $\frac{x}{5}$ + 500

Thus, the profit function P(x) is given by

P(x) = S(x) - C(x) = 5x - $\frac{x^2}{100} - \frac{x}{5}$ - 500

i.e. P(x) = $\frac{24}{5}x - \frac{x^2}{100}$ - 500

On differentiating both sides w.r.t. x, we get

P'(x) = $\frac{24}{5} - \frac{x}{50}$

Now, P'(x) = 0 gives x = 240.

Also, P'(x) = $\frac{-1}{50}$.

So, P'(240) = $\frac{-1}{50}$ < 0

Thus, x = 240 is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20.

(c) A is true but R is false.

Explanation:

Assertion is true because distinct elements in Z (domain) has distinct images in Z (codomain).

Reason is false because of: A $\rightarrow$ B is said to be surjective if every element of B has at least one pre-Image in A.

Section B

21. Let x = cos θ

$\therefore \theta = \cos^{-1} x$

$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta-1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$

$= 2\theta$

$= 2 \cos^{-1} x$

OR

Given $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$

$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$

$= \frac{\pi}{2} - \pi$

$= -\frac{\pi}{2}$

Therefore we have,

$\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$

22. At any instant t, let the length of each edge of the cube be x, V be its volume and S be its surface area. Then,

$\frac{dV}{dt} = 7\ cm^3/sec$ ... (given) ... (i)

Now, $V = x^3 \Rightarrow \frac{dV}{dt} = \frac{dV}{dx} \cdot \frac{dx}{dt}$

$\Rightarrow 7 = \frac{d}{dx}\left(x^3\right) \cdot \frac{dx}{dt}$ .... $[\because V = x^3]$

$\Rightarrow 3x^2 \cdot \frac{dx}{dt} = 7$

$\Rightarrow \frac{dx}{dt} = \frac{7}{3x^2}$

$\therefore S = 6x^2 \Rightarrow \frac{dS}{dt} = \frac{dS}{dx} \cdot \frac{dx}{dt}$

$= \frac{d}{dx}\left(6x^2\right) \cdot \frac{7}{3x^2}$

$= \left(12x \times \frac{7}{3x^2}\right) = \frac{28}{x}$

$\Rightarrow \left[\frac{dS}{dt}\right]_{x=12} = \left(\frac{28}{12}\right)$ cm^2/sec $= 2\frac{1}{3}\ cm^2/sec$

Hence, the surface area of the cube is increasing at the rate of $2\frac{1}{3}\ cm^2/sec$ at the instant when its edge is 12 cm.

23. Given: $f(x) = a(x + \sin x) + a$

$f'(x) = a(1 + \cos x)$

for (x) to be increasing we must have

$f'(x) > 0$

$\Rightarrow a(1 + \cos x) > 0 \ldots (1)$

We know

$-1 \leq \cos x \leq 1,\ \forall\ x \in R$

$\Rightarrow 0 \leq (1 + \cos x) \leq 2, \forall\ x \in R$

$\Rightarrow a \in (0, \infty) \Rightarrow a \geq 0$ f(x) is increasing on R.

OR

By definition of marginal revenue we have,

marginal revenue $(MR) = \frac{dR}{dx}$

$= \frac{d}{dx}\left(3x^2 + 36x + 5\right) = 6x + 36$

When x = 5, then

MR = 6(5) + 36 = 30 + 36 = 66

Hence, the required marginal revenue is Rs. 66.

More amount of money spent for the welfare of the employees with the increase in marginal revenue show affinity and care for employees.

24. Let $I = \int \frac{x \cos^{-1} x}{\sqrt{1-x^2}} dx$

Put, $t = \cos^{-1} x$

$\Rightarrow dt = \frac{-1}{\sqrt{1-x^2}} dx$

Also, cos t = x

Thus, we have

$I = -\int t \cot dt$

Now let us solve this by the 'by parts' method.

I = - [t sint -$\int$ sin t dt]

$\Rightarrow$ I = -[t sin t - cos t] + C

Substituting the value t = $\cos^{-1}$ x, we get

I = -[$\cos^{-1}$ x sin t + x] + c

$\Rightarrow$ I = -[$\cos^{-1}$ x $\sqrt{1-x^2}$ + x] + c

25. Given: $f(x)=\cos(2x+\frac{\pi}{4})$

$f'(x)=-2\sin(2x+\frac{\pi}{4})$

Now,

$x\in(\frac{3\pi}{8},\frac{7\pi}{8})$

$\Rightarrow\quad \frac{3\pi}{8}<x<\frac{7\pi}{8}$

$\Rightarrow\quad \frac{3\pi}{4}<2x<\frac{7\pi}{4}$

$\Rightarrow\quad \frac{\pi}{4}+\frac{3\pi}{4}<2x+\frac{\pi}{4}<\frac{7\pi}{4}+\frac{\pi}{4}$

$\Rightarrow\quad \pi<2x+\frac{\pi}{4}<2\pi$

$\Rightarrow\quad \sin(2x+\frac{\pi}{4})<0$

$\Rightarrow\quad -2\sin(2x+\frac{\pi}{4})>0$

$\Rightarrow\quad f'(x)>0$

Hence, f(x) is increasing on $(\frac{3\pi}{8},\frac{7\pi}{8})$

Section C

26. According to the question, $I=\int\frac{(3\sin\theta-2)\cos\theta}{5-\cos^2\theta-4\sin\theta}d\theta$

$=\int\frac{(3\sin\theta-2)\cos\theta}{5-\left(1-\sin^{2\theta}\right)-4\sin\theta}d\theta$

Put $\sin\theta=t\Rightarrow\cos\theta d\theta=dt$

$\therefore\quad I=\int\frac{3t-2}{5-(1-t^2)-4t}dt$

$=\int\frac{3t-2}{4+t^2-4t}dt=\int\frac{3t-2}{(t-2)^2}dt$

$=\int\frac{3t-6+4}{(t-2)^2}dt=\int\frac{3(t-2)+4}{(t-2)^2}dt$

$=\int\frac{3}{(t-2)}dt+\int\frac{4}{(t-2)^2}dt$

$=3\log|t-2|+\frac{4(t-2)^{-2+1}}{-2+1}+C$

$\left[\because\int x^n dx=\frac{x^{n+1}}{n+1}+C, n\neq-1\right]$

$=3\log|t-2|-\frac{4}{(t-2)}+C$

$=3\log|\sin\theta-2|-\frac{4}{(\sin\theta-2)}+C$

$[Put\ t=\sin\theta]$

27. Let E_1 and E_2 be the events that two headed coin and unbiased coin is chosen respectively. Also let E be the event that all 5 tosses are heads.

Therefore,we have,

$P(E_1)=\frac{2}{10}=\frac{1}{5}$, $P(E_2)=\frac{4}{5}$

$P(E|E_1)=1$, $P(E|E_2)={}^5C_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{5-5}=\frac{1}{32}$

By using Bayes' theorem

$P(E_1|E)=\frac{P(E|E_1)P(E_1)}{P(E|E_1)P(E_1)+P(E|E_2)P(E_2)}$

$\Rightarrow P(E_1|E)=\frac{1\times\frac{1}{5}}{1\times\frac{1}{5}+\frac{1}{32}\times\frac{4}{5}}=\frac{32}{36}=\frac{8}{9}$

This is the required probability.

28. According to the question , $I=\int_{-1}^{2}|x^3-x|\,dx$

We can observe that,

$$|x^3-x|=\begin{cases}(x^3-x), & \text{when } -1<x<0\\ -(x^3-x), & \text{when } 0\le x<1\\ (x^3-x), & \text{when } 1\le x<2\end{cases}$$

By Splitting the intervals , we get

$I = \int_{-1}^{0} |x^3 - x|\, dx + \int_0^1 |x^3 - x|\, dx + \int_1^2 |x^3 - x|\, dx$

$\therefore I = \int_{-1}^{0} \left(x^3 - x\right) dx + \int_0^1 -\left(x^3 - x\right) dx + \int_1^2 \left(x^3 - x\right) dx$

$= \int_{-1}^{0} \left(x^3 - x\right) dx - \int_0^1 \left(x^3 - x\right) dx + \int_1^2 \left(x^3 - x\right) dx$

$= \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_{-1}^{0} - \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^4}{4} - \frac{x^2}{2}\right]_1^2$

$= \left[0 - \left(\frac{1}{4} - \frac{1}{2}\right)\right] - \left[\left(\frac{1}{4} - \frac{1}{2}\right) - 0\right] + \left[\left(\frac{16}{4} - \frac{4}{2}\right) - \left(\frac{1}{4} - \frac{1}{2}\right)\right]$

$= -\frac{1}{4} + \frac{1}{2} - \frac{1}{4} + \frac{1}{2} + 4 - 2 - \frac{1}{4} + \frac{1}{2}$

$= -\frac{3}{4} + \frac{3}{2} + 2$

$= \frac{-3+6+8}{4}$

$= \frac{11}{4}$

$\therefore I = \frac{11}{4}$ sq units.

OR

Let, $I = \int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$

Put t = cos2x

dt = -2sin2xdx

$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2} \int \frac{dt}{(t-2)(1-t)}$

Using partial fractions,

$\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t}$ .... (1)

A(1 - t) + B(t - 2) = 1

Putting 1 - t = 0

t = 1

A(0) + B(1 - 2) = 1

B = -1

Putting t - 2 = 0

t = 2

A(1 - 2) + B(0) = 1

A = -1

From equation (1), we get,

$\frac{1}{(t-2)(1-t)} = \frac{-1}{t-2} + \frac{-1}{1-t}$

$\int \frac{1}{(t-2)(1-t)} dt = \int \frac{1}{2-t} dt + \int \frac{1}{t-1} dt$

$= -\log|2-t| + \log|t-1| + c$

$= \log|t-1| - \log|2-t| + c$

$= \log|\cos 2x - 1| - \log|2 - \cos 2x| + c$

29. The given differential equation is,

y (1 - x^2) $\frac{dy}{dx}$ = x (1 + y^2)

$\Rightarrow \frac{y}{1+y^2} dy = \frac{x}{1-x^2} dx$

Integrating both sides,

$\int \frac{y}{1+y^2} dy = \int \frac{x}{1-x^2} dx$

Substituting 1 + y^2 = t and 1 - x^2 = u

2y dy = dt and -2x dx = du

$\therefore \frac{1}{2} \int \frac{1}{t} dt = \frac{-1}{2} \int \frac{1}{u} du$

$\Rightarrow \frac{1}{2}$ log |t| = $-\frac{1}{2}$ log |u| + log C

$\Rightarrow \frac{1}{2}$ log |1 + y^2| = $-\frac{1}{2}$ log |1 - x^2| + log C

$\Rightarrow \frac{1}{2}$ log (|1 + y^2| |1 - x^2|) = 2 log C

$\Rightarrow$ (1 + y^2) (1 - x^2) = C^2

$\Rightarrow$ (1 + y^2) (1 - x^2) = C_1 ...(where $C_1 = C^2$)

Hence, (1 + y^2) (1 - x^2) = C_1 is the required solution.

OR

Given differential equation is,

$(1 + e^{2x})dy + (1 + y^2)e^x\, dx = 0$

Above equation may be written as

$\frac{dy}{1+y^2} = \frac{-e^x}{1+e^{2x}} dx$

On integrating both sides, we get

$\int \frac{dy}{1+y^2} = -\int \frac{e^x}{1+e^{2x}} dx$

On putting $e^x = t \Rightarrow e^x\, dx = dt$ in RHS, we get

$\tan^{-1} y = -\int \frac{1}{1+t^2} dt$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1} t + C$

$\Rightarrow \quad \tan^{-1} y = -\tan^{-1}(e^x) + C$ ...(i) [put $t = e^x$]

Also, given that y = 1, when x = 0.

On putting above values in Eq. (i), we get

$\tan^{-1}1 = -\tan^{-1}(e^0) + C$

$\Rightarrow \quad \tan^{-1} 1 = -\tan^{-1} 1 + C \quad [\because e^0 = 1]$

$\Rightarrow \quad 2\tan^{-1} 1 = C$

$\Rightarrow \quad 2\tan^{-1}\left(\tan\frac{\pi}{4}\right) = C$

$\Rightarrow \quad C = 2 \times \frac{\pi}{4} = \frac{\pi}{2}$

On putting $C = \frac{\pi}{2}$ in Eq. (i), we get

$\tan^{-1} y = -\tan^{-1} e^x + \frac{\pi}{2}$

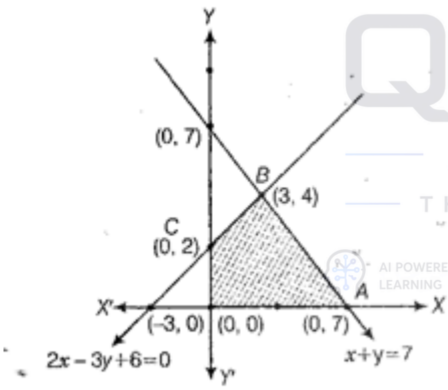
$\Rightarrow \quad y = \tan\left[\frac{\pi}{2} - \tan^{-1}(e^x)\right] = \cot\left[\tan^{-1}(e^x)\right]$

$= \cot\left[\cot^{-1}\left(\frac{1}{e^x}\right)\right] \left[\because \tan^{-1} x = \cot^{-1}\frac{1}{x}\right]$

$\therefore \quad y = \frac{1}{e^x}$

which is the required solution.

30. Minimise Z = 13x - 15y subject to the constraints: $x + y \leqslant 7, 2x - 3y + 6 \geqslant 0, x \geqslant 0, y \geqslant 0$.



replace (0,7) by (7,0) in horizontal line

Shaded region shown as OABC is bounded and coordinates of its corner points are (0, 0), (7, 0), (3, 4) and (0, 2), respectively.

Corner Points	**Corresponding value of Z**
(0, 0)	0
(7, 0)	91
(3, 4)	-21
(0, 2)	-30 (Minimum)

Hence, the minimum value of Z is -30 at (0,2).

OR

Subject to the constraints are

$4x + y \geq 80$

$x + 5y \geq 115$

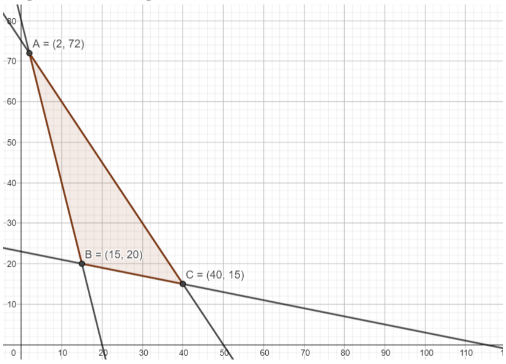
$3x + 2y \leq 150$

and the non negative constraint x,y ≥ 0

Converting the given inequations into equations, we get 4x + y = 80, x + 5y = 115, 3x + 2y = 150, x = 0 and y = 0

These lines are drawn on the graph and the shaded region ABC represents the feasible

region of the given LPP.



It can be observed that the feasible region is bounded. The coordinates of the corner

points of the feasible region are A(2, 72), B(15, 20) and C(40, 15) The values of the objective function, Z at these corner points are given in the following table:

Corner Point Value of the Objective Function Z = 6x + 3y

A(2, 72) : $Z = 6 \times 2 + 3 \times 72 = 228$

B(15, 20) : $Z = 6 \times 15 + 3 \times 20 = 150$

C(40, 15) : $Z = 6 \times 40 + 3 \times 15 = 285$

From the table, Z is minimum at x = 15 and y = 20 and the minimum value of Z is 150. Thus, the minimum value of Z is 150.

31. Let y = $x^{\log x}$ + $(\log x)^x$

Also, let u = $(\log x)^x$ and v = $x^{\log x}$

$\therefore$ y = v + u

$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} + \frac{du}{dx}$ ...(i)

Now, u = $(\log x)^x$

$\Rightarrow$ log u = log[$(\log x)^x$]

$\Rightarrow$ log u = x log(log x)

Differentiating both sides with respect to x,

$\frac{1}{u}\frac{du}{dx} = \log(\log x)\frac{d}{dx}(x) + x\frac{d}{dx}[\log(\log x)]$

$\Rightarrow \frac{du}{dx} = u\left[\log(\log x) + x\frac{1}{\log x}\frac{d}{dx}(\log x)\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{x}{\log x} \times \frac{1}{x}\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$...(ii)

Also, v = $x^{\log x}$

$\Rightarrow$ log v = log $x^{\log x}$

$\Rightarrow$ log v = log x log x = $(\log x)^2$

Differentiating both sides with respect to x,

$\frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left[(\log x)^2\right]$

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = 2(\log x)\frac{d}{dx}(\log x)$

$\Rightarrow \frac{dv}{dx} = 2v(\log x)\frac{1}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$

$\Rightarrow \frac{dv}{dx} = 2x^{\log x}\frac{\log x}{x}$ ....(iii)

From (i), (ii) and (iii), we obtain

$\frac{dy}{dx} = 2x^{\log x}\frac{\log x}{x} + (\log x)^x\left[\log(\log x) + \frac{1}{\log x}\right]$

Section D

32. According to the question ,

Given curves are

x - y + 2 = 0....(i)

x = $\sqrt{y}$...(ii)

Consider x = $\sqrt{y} \Rightarrow x^2 = y$, which represents the parabola

vertex of parabola is (0, 0)

axis of parabola is Y-axis.

Now, the point of intersection of Eqs.(i) and (ii) is given by

x = $\sqrt{x+2}$

Squaring on both sides ,

$\Rightarrow x^2 = x + 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = -1, 2$

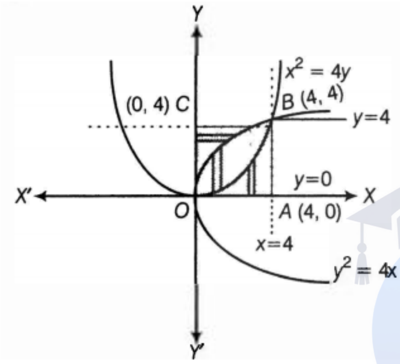
When x = -1 , does not satisfy the Eq. (ii).

When x = 2 , then $2 = \sqrt{y} \Rightarrow y = 4$

Hence, the point of intersection is (2, 4).

But actual equation of given parabola is x = $\sqrt{y}$, it means a semi-parabola which is on right side of Y -axis.

The graph of given curves are shown below:



Clearly, area of bounded region = Area of region OABO

$= \int_0^2 \left[y_{(\text{ line })} - y_{(\text{ parabola })}\right] dx$

$= \int_0^2 (x+2)dx - \int_0^2 x^2 dx$

$= \left[\frac{x^2}{2} + 2x\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2$

$= \left[\frac{4}{2} + 4 - 0\right] - \left[\frac{8}{3} - 0\right]$

$= 6 - \frac{8}{3}$

$= \frac{18-8}{3}$

$= \frac{10}{3}$ sq.units.

33. i. x is greater than y, x, y $\in$ N

For xRx x > x is not true for any x $\in$ N.

Therefore, R is not reflexive.

Let (x, y) $\in$ R $\Rightarrow$ xRy

x > y

but y > x is not true for any x, y $\in$ N

Thus, R is not symmetric.

Let xRy and yRz

x > y and y > z $\Rightarrow$ x > z

$\Rightarrow$ xRz

So, R is transitive.

ii. x + y = 10, x, y $\in$ N

R={(x,y) : x+y = 10 ,x,y $\in$ R}

R = {(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)} (1, 1) $\notin$ R

So, R is not reflexive.

(x, y) $\in$ R $\Rightarrow$ (y, x) $\in$ R

Therefore, R is symmetric.

$(1, 9) \in R$, $(9, 1) \in R \Rightarrow (1, 1) \notin R$

Hence, R is not transitive.

iii. Given xy, is square of an integer x, y $\in$ N

$\Rightarrow$ R = {(x, y) : xy is a square of an integer x, y $\in$ N}

$(x, x) \in R, \forall x \in N$

As x^2 is square of an integer for any $x \in N$

Hence, R is reflexive.

If $(x, y) \in R \Rightarrow (y, x) \in R$

Therefore, R is symmetric.

If $(x, y) \in R$ $(y, z) \in R$

So, xy is square of an integer and yz is square of an integer.

Let xy = m^2 and yz = n^2 for some m, n $\in$ Z

$x = \frac{m^2}{y}$ and $z = \frac{x^2}{y}$

$xz = \frac{m^2 n^2}{y^2}$, Which is square of an integer.

So, R is transitive.

iv. x + 4y = 10, x, y $\in$ N

R = {(x, y) : x + 4y = 10, x, y $\in$ N}

R{(2, 2), (6, 1)}

(1, 1), (3, 3) ..... $\notin$ R

Thus, R is not reflexive.

$(6, 1) \in R$ but $(1, 6) \notin R$

Hence, R is not symmetric.

$(x, y) \in R \Rightarrow x + 4y = 10$ but $(y, z) \in R$

$y + 4z = 10 \Rightarrow (x, z) \in R$

So, R is transitive.

OR

A = R - {3}, B = R - {1}

f : A $\rightarrow$ B is defined as $f(x) = \left(\frac{x-2}{x-3}\right)$.

Let x, y $\in$ A such that f(x) = f(y).

$\Rightarrow \frac{x-2}{x-3} = \frac{y-2}{y-3}$

$\Rightarrow$ (x - 2) (y - 3) = (y - 2) (x - 3)

$\Rightarrow$ xy - 3x - 2y + 6 = xy - 3y - 2x + 6

$\Rightarrow$ -3x - 2y = -3y - 2x

$\Rightarrow$ 3x - 2x = 3y - 2y

$\Rightarrow$ x = y

Therefore, f is one-one.

Let y $\in$ B = R - {1}

Then, $y \neq 1$.

The function f is onto if there exists x $\in$ A such that f(x) = y.

Now, f (x) = y

$\Rightarrow \frac{x-2}{x-3} = y$

$\Rightarrow$ x - 2 = xy - 3y

$\Rightarrow$ x(1 - y) = -3y + 2

$\Rightarrow x = \frac{2-3y}{1-y} \in \mathbf{A} \qquad [y \neq 1]$

Thus, for any y $\in$ B, there exists $\frac{2-3y}{1-y} \in \mathrm{A}$ such that

$$f\left(\frac{2-3y}{1-y}\right) = \frac{\left(\frac{2-3y}{1-y}\right)-2}{\left(\frac{2-3y}{1-y}\right)-3} = \frac{2-3y-2+2y}{2-3y-3+3y} = \frac{-y}{-1} = y$$

$\therefore$ f is onto.

Hence, function f is one-one and onto.

34. $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

$\therefore$ |A| = 2(-4 + 4) + 3(-6 +4)+ 5(3 - 2) = 0 -6 +5 = -1

Now, $A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$

$A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$

$A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$

$\therefore A^{-1} = \frac{1}{|A|}(adjA) = -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ ....(1)

Now, the given system of equations can be written in the form of AX = B, where

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

The solution of the system of equations is given by $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \text{ [Using (1)]}$$

$$= \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x = 1, y = 2 and z = 3.

35. $\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{a}_2 - \vec{a}_1 = \hat{i} - \hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$

$= 3\hat{i} - \hat{j} - 7\hat{k}$

$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{9+1+49} = \sqrt{59}$

Also, $(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1) = (3\hat{i} - \hat{j} - 7\hat{k})(\hat{i} - \hat{k}) = 3 + 7 + 0 = 10$

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2).(\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right| = \frac{10}{\sqrt{59}}$$

OR

Let P be the point with position vector $\vec{p} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and M be the image of P in the plane $\vec{r} \cdot (2\hat{\imath} - \hat{\jmath} + \hat{k}) = 4$.

In addition, let Q be the foot of the perpendicular from P on to the given plane. So, Q is the midpoint of PM.

Direction ratios of PM are proportional to 2, –1, 1 as PM is normal to the plane and parallel to $2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Recall the vector equation of the line passing through the point with position vector $\vec{r}$ and parallel to vector $\vec{b}$ is given by

$\vec{r} = \vec{a} + \lambda\vec{b}$

Here, $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$

Hence, the equation of PM is

$\vec{r} = (3\hat{i} + \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

$\therefore \vec{r} = (3+2\lambda)\hat{i} + (1-\lambda)\hat{j} + (2+\lambda)\hat{k}$

Let the position vector of M be $\vec{m}$. As M is a point on this line, for some scalar α, we have

$\Rightarrow \vec{m} = (3+2\alpha)\hat{i} + (1-\alpha)\hat{j} + (2+\alpha)\hat{k}$

Now, let us find the position vector of Q, the midpoint of PM.

Let this be $\vec{q}$.

Using the midpoint formula, we have

$\vec{q} = \frac{\vec{p}+\vec{m}}{2}$

$\Rightarrow \vec{q} = \frac{[3\hat{i}+\hat{j}+2\hat{k}]+[(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}]}{2}$

$\Rightarrow \vec{q} = \frac{(3+(3+2\alpha))\hat{i}+(1+(1-\alpha))\hat{j}+(2+(2+\alpha))\hat{k}}{2}$

$\Rightarrow \vec{q} = \frac{(6+2\alpha)\hat{i}+(2-\alpha)\hat{j}+(4+\alpha)\hat{k}}{2}$

$\therefore \vec{q} = (3+\alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}$

This point lies on the given plane, which means this point satisfies the plane equation $\vec{r}\cdot(2\hat{\imath}-\hat{\jmath}+\hat{k})=4$.

$\Rightarrow \left[(3+\alpha)\hat{i} + \frac{(2-\alpha)}{2}\hat{j} + \frac{(4+\alpha)}{2}\hat{k}\right]\cdot(2\hat{\imath}-\hat{j}+\hat{k})=4$

$\Rightarrow 2(3+\alpha)-\left(\frac{2-\alpha}{2}\right)(1)+\left(\frac{4+\alpha}{2}\right)(1)=4$

$\Rightarrow 6+2\alpha+\frac{4+\alpha-(2-\alpha)}{2}=4$

$\Rightarrow 2\alpha+(1+\alpha)=-2$

$\Rightarrow 3\alpha=-3$

$\therefore \alpha=-1$

We have the image $\vec{m}=(3+2\alpha)\hat{i}+(1-\alpha)\hat{j}+(2+\alpha)\hat{k}$

$\Rightarrow \vec{m}=[3+2(-1)]\hat{i}+[1-(-1)]\hat{j}+[2+(-1)]\hat{k}$

$\therefore \vec{m}=\hat{\imath}+2\hat{\jmath}+\hat{k}$

Therefore, the image is (1, 2, 1)

The foot of the perpendicular $\vec{q}=(3+\alpha)\hat{i}+\frac{(2-\alpha)}{2}\hat{j}+\frac{(4+\alpha)}{2}\hat{k}$

$\Rightarrow \vec{q}=[3+(-1)]\hat{i}+\left[\frac{2-(-1)}{2}\right]\hat{j}+\left[\frac{4+(-1)}{2}\right]\hat{k}$

$\therefore \vec{q}=2\hat{i}+\frac{3}{2}\hat{j}+\frac{3}{2}\hat{k}$

Thus, the position vector of the image is $\hat{\imath}+2\hat{\jmath}+\hat{k}$ and that of the foot of the perpendicular is $2\hat{\imath}+\frac{3}{2}\hat{\jmath}+\frac{3}{2}\hat{k}$

Section E

36. i. It is given that if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 0.3

ii. It is given that, if India loose any match, then the probability that it wins the next match is 0.3.

$\therefore$ Required probability = 1 - 0.3 = 0.7

iii. Required probability = P(India losing first match) $\cdot$ P(India losing second match when India has already lost first match) = 0.4 $\times$ 0.7 = 0.28

OR

Required probability = P(India winning first match) $\cdot$ P(India winning second match if India has already won first match) $\cdot$ P(India winning third match if India has already won first two matches) = 0.6 $\times$ 0.4 $\times$ 0.4 = 0.096

37. i. Here,

Position vector of A is $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$

Position vector of B is $\vec{b}=3\hat{i}-3\hat{j}-2\hat{k}$

Position vector of C is $\vec{c}=-2\hat{i}+2\hat{j}+6\hat{k}$

$\therefore \vec{a}+\vec{b}+\vec{c}$ = (1 + 3 - 2)$\hat{i}$ + (4 - 3 + 2)$\hat{j}$ + (2 - 2 + 6)$\hat{k}$

= 2$\hat{i}$ + 3$\hat{j}$ + 6$\hat{k}$

Thus, $|\vec{a}+\vec{b}+\vec{c}|$ = $\left|\sqrt{(2)^2+(3)^2+(6)^2}\right|$

= $|\sqrt{4+9+16}|$

= $\sqrt{29}$

ii. Given, $\vec{a}=4\hat{i}+6\hat{j}+12\hat{k}$,

$|\vec{a}|=\sqrt{4^2+6^2+12^2}$ = 14

Therefore, the unit vector in direction of $\vec{a}$ is given by

$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{4\hat{i}+6\hat{j}+12\hat{k}}{14}$

= $\frac{4}{14}\hat{i}+\frac{6}{14}\hat{j}+\frac{12}{14}\hat{k}$

= $\frac{2}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{6}{7}\hat{k}$

iii. We have, A(1, 4, 2), B(3, -3, -2) and C(-2, 2, 6)

Now, $\overrightarrow{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$

and $\overrightarrow{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$

$\therefore \overrightarrow{AB} \times \overrightarrow{AC}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$

= $\hat{i}(-28-8) - \hat{j}(8-12) + \hat{k}(-4-21)$

= - $36\hat{i} + 4\hat{j} - 25\hat{k}$

Now, $|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$

= $|\sqrt{1296+16+625}| = \sqrt{1937}$

$\therefore$ Area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

= $\frac{1}{2}\sqrt{1937}$ sq. units

OR

Triangle law of addition for $\triangle$ABC is given by

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$

If the given points lie on the straight line, then the points will be collinear and so area of $\triangle$ABC = 0

Then, $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ = 0.

Also, if a, b, c are the position vector of the three vertices A, B and C of $\triangle$ABC, then area of triangle is $\frac{1}{2}|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$.

38. i. f(x) = $-0.1x^2$ + mx + 98.6, being a polynomial function, is differentiable everywhere, hence, differentiable in (0, 12).

ii. f(x) = -0.2x + m

At Critical point

0 = -0.2 $\times$ 6 + m

m = 1.2

iii. f(x) = $-0.1x^2$ + 1.2x + 98.6

f'(x) = -0.2x + 1.2 = -0.2(x - 6)

In the Interval	f'(x)	Conclusion
(0, 6)	+Ve	f is strictly increasing in [0, 6]
(6, 12)	-Ve	f is strictly decreasing in [6, 12]

OR

f(x) = $-0.1x^2$ + 1.2x + 98.6,

f'(x) = -0.2x + 1.2, f'(6) = 0,

f''(x) = -0.2

f''(6) = -0.2 < 0

Hence, by second derivative test 6 is a point of local maximum. The local maximum value = f(6) = -0.1 $\times$ 6^2 + 1.2 $\times$ 6 + 98.6 = 102.2

We have f(0) = 98.6, f(6) = 102.2, f(12) = 98.6

6 is the point of absolute maximum and the absolute maximum value of the function = 102.2.

0 and 12 both are the points of absolute minimum and the absolute minimum value of the function = 98.6.