Section A

1. **(a)** A is a square matrix

Explanation:

A square matrix is a null matrix if all its entries are zero.

2.

(b) 27

Explanation:

Since the matrix is of order 3 so 3 will be taken common from each row or column.

So, k = 27

3.

(d) $\tan^{-1}\left(\frac{y}{x}\right) = \log x + C$

Explanation:

We have,

$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

$\frac{dy}{dx} = 1 + \frac{y}{x} + \frac{y^2}{x^2}$ ...(i)

Let y = vx

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$1 + v + v^2 = v + x\frac{dv}{dx}$ ....from(i)

$1 + v^2 = x\frac{dv}{dx}$

$\frac{dx}{x} = \frac{dv}{1+v^2}$

$\int \frac{dx}{x} = \int \frac{dv}{1+v^2}$

log |x| = $\tan^{-1}$x + c

4. **(a)** discontinuous at exactly three points

Explanation:

We have, $f(x) = \frac{4-x^2}{4x-x^3} = \frac{(4-x^2)}{x(4-x^2)}$

$= \frac{(4-x^2)}{x(2^2-x^2)} = \frac{4-x^2}{x(2+x)(2-x)}$

Clearly, f(x) is discontinuous at exactly three points x = 0, x = -2 and x = 2.

5.

(c) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

Explanation:

$13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$

If a line makes angles α, β and γ with the axis, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ = 1 ....(i)

Let r be the length of the line segment. Then,

$r\cos\alpha = 12, r\cos\beta = 4, r\cos\gamma = 3$ ....(ii)

$\Rightarrow (r\cos\alpha)^2 + (r\cos\beta)^2 + (r\cos\gamma)^2$ = $12^2 + 4^2 + 3^2$

$\Rightarrow r^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$ = 169

$\Rightarrow r^2$ (1) = 169 [From (i)]

$\Rightarrow r = \sqrt{169}$

$\Rightarrow$ r = $\pm$13

$\Rightarrow$ r = 13 (since length cannot be negative)

Substituting r = 13 in (ii)

We get,
$\cos\alpha = \frac{12}{13}, \cos\beta = \frac{4}{13}, \cos\gamma = \frac{1}{13}$
Thus, the direction cosines of the line are
$\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$

6.

(d) not defined
Explanation:
not defined

7. **(a)** a feasible region
Explanation:
a feasible region

8.

(b) 1
Explanation:
$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$
= $\sin\left(\frac{\pi}{3} + \frac{\pi}{6}\right)$ = $\sin\left(\frac{\pi}{2}\right)$ = 1

9.

(c) $\frac{3}{2}x^{\frac{2}{3}} + C$
Explanation:
Given:
$\int \frac{1}{\sqrt[3]{x}} dx$
$\int x^n dx = \frac{x^{n+1}}{n+1} + c$
$\int \frac{1}{\sqrt[3]{x}} dx = \frac{x^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} + c$
$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$
$= \frac{3}{2}x^{\frac{2}{3}} + c$

10.

(d) $\frac{\pi}{3}$
Explanation:
Given the matrix $A = \begin{bmatrix} \tan x & 1 \\ -1 & \tan x \end{bmatrix}$, we find its transpose $A' = \begin{bmatrix} \tan x & -1 \\ 1 & \tan x \end{bmatrix}$ Setting $A + A' = 2\sqrt{3}I$ leads to $2\tan x = 2\sqrt{3}$, giving $\tan x = \sqrt{3}$. Thus, $x = \frac{\pi}{3}$.

11.

(c) at an infinite number of points
Explanation:
First, we will convert the given inequations into equations, we obtain the following equations:
3x + 4y = 24, 8x + 6y = 48, x = 5, y = 6, x = 0 and y = 0
The line 3x + 4y = 24 meets the coordinate axis at A(8, 0) and B(0, 6). Join these points to obtain the line 3x + 4y = 24 Clearly, (0, 0) satisfies the inequation 3x + 4y $\leq$ 24 . So, the region in x y -plane that contains the origin represents the solution set of the given equation. The line 8x + 6y = 48 meets the coordinate axis at C(6, 0) and D(0, 8) . Join these points to obtain the line 8x + 6y = 48 Clearly, (0, 0) satisfies the inequation 8x + 6y $\leq$ 48 . So, the region in x y -plane that contains the origin represents the solution set of the given equation.
x = 5 is the line passing through x = 5 parallel to the Y axis.

QuestIQ ACADEMY THINK BEYOND LIMITS. AI POWERED LEARNING CONCEPT MASTERY PERFORMANCE FOCUSED EXCELLENCE DRIVEN Powered by QualCrypt

y = 6 is the line passing through y = 6 parallel to the X axis.

The region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations.

These lines are drawn using a suitable scale.

The corner points of the feasible region are O(0, 0), G(5, 0), $F\left(5, \frac{4}{3}\right)$, $E\left(\frac{24}{7}, \frac{24}{7}\right)$ and B(0, 6)

The values of Z at these corner points are as follows:

Corner point : Z = 4x + 3y

O(0, 0) : $4 \times 0 + 3 \times 0 = 0$

G(5, 0) : $4 \times 5 + 3 \times 0 = 20$

$F\left(5, \frac{4}{3}\right)$: $4 \times 5 + 3 \times \frac{4}{3} = 24$

$E\left(\frac{24}{7}, \frac{24}{7}\right)$: $4 \times \frac{24}{7} + 3 \times \frac{24}{7} = \frac{196}{7} = 28$

B(0, 6) : $4 \times 0 + 3 \times 6 = 18$

We see that the maximum value of the objective function z is 24 which is at $F\left(5, \frac{4}{3}\right)$ and $E\left(\frac{24}{7}, \frac{24}{7}\right)$ Thus, the optimal value of Z is 24

As, we know that if an LPP has two optimal solutions, then there are an infinite number of optimal solutions. Therefore, the given objective function can be subjected at an infinite number of points.

12. **(a)** f(x) and g(x) both are continuous at x = 0

Explanation:

Given f(x) = |x| and g(x) = $|x^3|$,

$$f(x) = \begin{cases} -x, & x \leq 0 \\ x, & x > 0 \end{cases}$$

Checking differentiability and continuity,

LHL at x =0,

$$\lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(0-h) = 0$$

RHL at x =0,

$$\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} (0+h) = 0$$

And f(0)=0

Hence, f(x) is continuous at x =0.

LHD at x =0,

$$\lim_{x \to 0^-} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0-h)-f(0)}{0-h-0}$$

$$= \lim_{h \to 0} \frac{(0-h)-(0)}{-h} = -1$$

RHD at x =0,

$$\lim_{x \to 0^+} \frac{f(x)-f(0)}{x-0} = \lim_{h \to 0} \frac{f(0+h)-f(0)}{0+h-0}$$

$$= \lim_{h \to 0} \frac{(0+h)-(0)}{h} = 1$$

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x =0.

$$g(x) = \begin{cases} -x^3, & x \leq 0 \\ x^3, & x > 0 \end{cases}$$

Checking differentiability and continuity,

LHL at x =0,

$$\lim_{x \to 0^-} g(x) = \lim_{h \to 0} g(0-h) = \lim_{h \to 0} -(0-h)^3 = 0$$

RHL at x =0,

$$\lim_{x \to 0^+} g(x) = \lim_{h \to 0} g(0+h) = \lim_{h \to 0} (0+h)^3 = 0$$

And g(0)=0

Hence, g(x) is continuous at x =0.

LHD at x =0,

$$\lim_{x\to 0^-}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0-h)-g(0)}{0-h-0}$$

$$=\lim_{h\to 0}\frac{(0-h)^3-(0)}{-h}=0$$

RHD at x =0,

$$\lim_{x\to 0^+}\frac{g(x)-g(0)}{x-0}=\lim_{h\to 0}\frac{g(0+h)-g(0)}{0+h-0}$$

$$=\lim_{h\to 0}\frac{(0+h)^3-(0)}{h}=0$$

∵ LHD = RHD

∴ g(x) is differentiable at x =0.

13.

(d) $\vec{r}=(5\hat{i}-2\hat{j}+4\hat{k})\ +\lambda(2\hat{i}-\hat{j}+3\hat{k})$

Explanation:

Fixed point is $5\hat{\imath}-2\hat{\jmath}+4\hat{k}$ and parallel vector is $2\hat{\imath}-\hat{\jmath}+3\hat{k}$

Equation $\vec{r}=5\hat{\imath}-2\hat{\jmath}+4\hat{k}+\lambda(2\hat{\imath}-\hat{\jmath}+3\hat{k})$

14.

(c) $\frac{5}{8}$

Explanation:

$P(\bar{A})=\frac{1}{2}, P(\bar{B})=\frac{2}{3}$ and $P(A\cap B)=\frac{1}{4}$

$\Rightarrow P(A)=\frac{1}{2}; P(B)=\frac{1}{3}$

We have, $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

$=\frac{7}{12}$

$P\left(\frac{\bar{A}}{\bar{B}}\right)=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$

$=\frac{P(\overline{A\cup B})}{P(\bar{B})}$

$=\frac{1-P(A\cup B)}{P(\bar{B})}$

$=\frac{1-\frac{7}{12}}{\frac{2}{3}}$

$=\frac{5}{8}$

15.

(c) y″ + (y′)^2 + y = 0

Explanation:

The highest order derivate is y" which is second order.

16.

(d) $\mathbb{R}$

Explanation:

$\mathbb{R}$

17.

(c) $-\frac{1}{t^2}$

Explanation:

To find $\frac{dy}{dx}$, we can use the chain rule:

$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$

First, let's find $\frac{dy}{dt}$:

$\frac{dy}{dt}=-\frac{a}{t^2}$

Next, let's find $\frac{dt}{dx}$:

$\frac{dt}{dx} = \frac{1}{a}$

Substituting these values into the chain rule equation:

$\frac{dy}{dx} = \left(-\frac{a}{t^2}\right) \cdot \left(\frac{1}{a}\right)$

Simplifying:

$\frac{dy}{dx} = -\frac{1}{t^2}$

Therefore, the correct answer is $-\frac{1}{t^2}$.

18.

(d) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$

Explanation:

Direction cosines of the line, whose direction ratios are 2, 3, -6 are:

$\frac{2}{\sqrt{2^2+3^2+(-6)^2}}, \frac{3}{\sqrt{2^2+3^2+(-6)^2}}, \frac{-6}{\sqrt{2^2+3^2+(-6)^2}}$

or $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$

Since, line makes obtuse angle with Y-axis, then $\cos\beta < 0$

Therefore, direction cosines are $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$.

19.

(d) A is false but R is true.

Explanation:

Assertion (A) is wrong.

Since, direction cosines of a line are the cosines of the angles made by the line with the positive directions of the coordinate axes.

Reason (R) is correct.

Since, the slope of the line x - 2 = 0 is ∞.

The slope of line $\sqrt{3}x$ - y - 2 = 0 is $\sqrt{3}$.

Let $m_1 = \infty$, $m_2 = \sqrt{3}$ and the angle between the given lines is θ.

$\Rightarrow \tan\theta - \left|\frac{m_2-m_1}{1+m_1 \cdot m_2}\right|$

$\Rightarrow \tan\theta = \left|\frac{\frac{m_2}{m_1}-1}{\frac{1}{m_1}+m_2}\right|$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30^o$

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

We have, I = $\int xe^{x^2} dx$

Put x^2 = t $\Rightarrow$ 2xdx = dt

$\therefore$ I = $\frac{1}{2}\int e^t dt$

$= \frac{1}{2}e^t + C$

$= \frac{1}{2}e^{x^2} + C$

Section B

21. Let (x, y) be any point on the line containing (3, 1) and (9, 3),then the required equation is,

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1 , we get,

$x[1-3] - y[3-9] + 1[9-9] = 0$

$\Rightarrow -2x + 6y = 0$

$x = 3y$ which is the required equation of the line.

QuestIQ ACADEMY
THINK BEYOND LIMITS.
AI POWERED LEARNING
CONCEPT MASTERY
PERFORMANCE FOCUSED
EXCELLENCE DRIVEN
Powered by QualCrypt

OR

By using the definition, on expanding the given determinant with respect to C_1

$$D = \begin{vmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ -2 & 1 & -3 \end{vmatrix}$$

$$\Rightarrow D = (-1)^{1+1}(2)\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} + (-1)^{2+1}(1)\begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} + (-1)^{3+1}(-2)\begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$$

$$\Rightarrow D = 2\begin{vmatrix} 2 & 3 \\ 1 & -3 \end{vmatrix} - \begin{vmatrix} 3 & -2 \\ 1 & -3 \end{vmatrix} - 2\begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix}$$

$\Rightarrow D = 2(-6-3) - (-9+2) - 2(9+4) = -18 + 7 - 26 = -37$

22. The given integral is $\int_1^3 \frac{\cos(\log x)}{x} dx$

Let log x = t

Differentiating w.r.t. x, we get

$\frac{1}{x} dx = dt$

Now,

$x = 0 \Rightarrow t = 0$

$x = 3 \Rightarrow t = \log 3$

$\int_1^3 \frac{\cos(\log x)}{x} dx$

$= \int_0^{\log 3} \cos t dt \ [\because \int \cos t = \sin t]$

$= [\sin t]_0^{\log 3}$

=sin (log 3) - sin 0

$= \sin(\log 3)$

$\int_1^3 \frac{\cos(\log x)}{x} dx = \sin(\log 3)$

23. Let x be the length of an edge of the cube, V be the volume and S be the surface area at any time t.

Then, $V = x^3$ and $S = 6x^2$.

Given

$\frac{dV}{dt} = 8 \mathrm{cm}^3/\mathrm{sec}$

$\Rightarrow \frac{d}{dt}(x^3) = 8 \Rightarrow 3x^2 \frac{dx}{dt} = 8 \Rightarrow \frac{dx}{dt} = \frac{8}{3x^2}$

Now, $S = 6x^2$

$\Rightarrow \quad \frac{dS}{dt} = 12x \frac{dx}{dt}$

$\Rightarrow \quad \frac{dS}{dt} = 12x \times \frac{8}{3x^2}$

$\Rightarrow \quad \frac{dS}{dt} = \frac{32}{x}$

$\Rightarrow \quad \left(\frac{dS}{dt}\right)_{x=12} = \frac{32}{12} \mathrm{cm}^2/\mathrm{sec} = \frac{8}{3} \mathrm{cm}^2/\mathrm{sec}$

OR

Given: $f(x) = (x+2)e^{-x}$

$f'(x) = e^{-x} - e^{-x}(x+2)$

$= e^{-x}(1 - x - 2)$

$= -e^{-x}(x+1)$

For Critical points

$f'(x) = 0$

$\Rightarrow -e^{-x}(x+1) = 0$

$\Rightarrow x = -1$

Clearly $f'(x) > 0$ if $x < -1$

$f'(x) < 0$ if $x > -1$

Hence f(x) increases in $(-\infty, -1)$, decreases in $(-1, \infty)$

24. When a die is thrown, the sample space is

S = {1, 2, 3, 4, 5, 6}

$\Rightarrow n(S) = 6$

Also, A : number is even and B : number Is red

$\therefore A = \{2, 4, 6\}$ and $B = \{1, 2, 3\}$ and $A \cap B = \{2\}$

$\Rightarrow$ n(A) = 3, n (B) = 3 and n(A∩B) = 1

Now, P(A) = $\frac{n(A)}{n(S)}=\frac{3}{6}=\frac{1}{2}$

P(B) =$\frac{n(B)}{n(S)}=\frac{3}{6}=\frac{1}{2}$

and P(A∩B) = $\frac{n(A\cap B)}{n(S)}=\frac{1}{6}$

Now, P(A) × P(B) = $\frac{1}{2}\times\frac{1}{2}$

$=\frac{1}{4}\neq\frac{1}{6}=P(A\cap B)$

$\therefore \quad P(A\cap B)\neq P(A)\times P(B)$

Thus, A and B are not independent events.

25. $\because$ highest derivative is $\frac{dy}{dx}$. so order is 1.

And degree is highest power of highest derivative so degree is 2.

Section C

26. Let $I=\int\frac{x^3+1}{x^3-x}dx=\int\frac{(x+1)(x^2-x+1)}{x(x^2-1)}dx$

$=\int\frac{(x^2-x+1)}{x(x-1)}dx=\int\left[1+\frac{1}{x(x-1)}\right]dx$

Consider, $\frac{1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$

$\Rightarrow\frac{1}{x(x-1)}=\frac{A(x-1)+Bx}{x(x-1)}$

$\Rightarrow 1=A(x-1)+Bx$

On solving, we get $A=-1, B=1$

$\Rightarrow\frac{1}{x(x-1)}=\frac{-1}{x}+\frac{1}{x-1}$

$\therefore I=\int\left(1+\frac{1}{x-1}-\frac{1}{x}\right)dx=x+\log|x-1|-\log|x|+c$

27. Given f(x) = $\begin{cases}\lambda\left(x^2+2\right), \text{ if } x\leq 0\\ 4x+6, \text{ if } x>0\end{cases}$ is continuous at x = 0

Since f(x) is continuous at x=0.Therefore, we have,

LHL = RHL = f(0).......(i)

Here, RHL = $\lim_{x\to 0^+}f(x)=\lim_{x\to 0^+}(4x+6)$

$=\lim_{h\to 0}[4(0+h)+6]$

$=\lim_{h\to 0}(4h+6)$

= 4 × 0 + 6 = 6

From Eq(i), RHL = f(0)

$\Rightarrow \quad 2\lambda=6\Rightarrow\lambda=3$

Now, given function becomes

f(x) = $\begin{cases}3\left(x^2+2\right), \text{ if } x\leq 0\\ 4x+6, \text{ if } x>0\end{cases}$

Now, let us check the differentiability at x = 0.

LHD = $\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$

$=\lim_{h\to 0}\frac{3\left[(0-h)^2+2\right]-3(0+2)}{-h}$

$=\lim_{h\to 0}\frac{3\left[h^2+2\right]-6}{-h}=\lim_{h\to 0}(-3h)$ = 0

and RHD = $\lim_{h\to 0}\frac{f(0+h)-f(0)}{h}$

$=\lim_{h\to 0}\frac{[4(0+h)+6]-3(0+2)}{h}=\lim_{h\to 0}\frac{4h}{h}$ = 4

$\because$ LHD $\neq$ RHD

$\therefore$ f(x) is not differentiable at x = 0.

28. $I=\int_0^{\pi/2}\frac{\cos^2x}{\cos^2x+4\sin^2x}dx$

$=\int_0^{\pi/2}\frac{\cos^2x}{\cos^2x+4(1-\cos^2x)}dx$

$=\int_0^{\pi/2}\frac{\cos^2x}{\cos^2x+4-4\cos^2x}dx$

QuestIQ ACADEMY THINK BEYOND LIMITS. AI POWERED LEARNING CONCEPT MASTERY PERFORMANCE FOCUSED EXCELLENCE DRIVEN Powered by QualCrypt

$= \int_0^{\pi/2} \frac{\cos^2 x}{4-3\cos^2 x} dx$
$= \frac{-1}{3} \int_0^{\pi/2} \frac{-3\cos^2 x}{4-3\cos^2 x} dx$
$= \frac{-1}{3} \int_0^{\pi/2} \frac{4-3\cos^2 x-4}{4-3\cos^2 x} dx$
$= \frac{-1}{3} \int_0^{\pi/2} \left(1 - \frac{4}{4-3\cos^2 x}\right) dx$
$= \frac{-1}{3} \int_0^{\pi/2} 1 dx + \frac{4}{3} \int_0^{\pi/2} \frac{dx}{4-3\cos^2 x}$
$= \frac{-1}{3} [x]_0^{\pi/2} + \frac{4}{3} \int_0^{\pi/2} \frac{dx/\cos^2 x}{\frac{4}{\cos^2 x} - \frac{3\cos^2 x}{\cos^2 x}}$
$= \frac{-1}{3} \left[\frac{\pi}{2} - 0\right] + \frac{4}{3} \int_0^{\pi/2} \frac{\sec^2 x dx}{4\sec^2 x-3}$
$= \frac{-\pi}{6} + \frac{4}{3} \int_0^{\pi/2} \frac{\sec^2 x dx}{4(1+\tan^2 x)-3} dx$
Put tan x = t
$\sec^2 x \, dx = dt$
$\therefore I = \frac{-\pi}{6} + \frac{4}{3} \int_0^{\infty} \frac{dt}{4(1+t^2)-3}$
$= \frac{-\pi}{6} + \frac{4}{3} \int_0^{\infty} \frac{dt}{4t^2+1}$
$= \frac{-\pi}{6} + \frac{1}{4} \times \frac{4}{3} \int_0^{\infty} \frac{dt}{t^2+\frac{1}{4}}$
$= \frac{-\pi}{6} + \frac{1}{3} \times \frac{1}{1/2} \left[\tan^{-1} \frac{t}{1/2}\right]_0^{\infty}$
$= \frac{-\pi}{6} + \frac{2}{3} \left[\tan^{-1} 2t\right]_0^{\infty}$
$= \frac{-\pi}{6} + \frac{2}{3} \left[\tan^{-1}\infty - \tan^{-1} 0\right]$
$= \frac{-\pi}{6} + \frac{2}{3} \left(\frac{\pi}{2} - 0\right)$
$= \frac{-\pi}{6} + \frac{\pi}{3}$
$= \pi/6$

OR

Let I = $= \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$
= $\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)} dx$
By using the properties of definite integrals
= $\int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx - I$
$\Rightarrow$ 2I $= \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$
Put cos x = t for $x = \pi \Rightarrow t = -1, x = 0 \Rightarrow t = +1$ and $-\sin x dx = dt$
Therefore 2I $= \pi \int_1^{-1} \frac{-dt}{1+t^2} = \pi \int_{-1}^{1} \frac{dt}{1+t^2}$
= $\pi \left[\tan^{-1} t\right]_{-1}^{1} = \pi \left[\tan^{-1}(+1) - \tan^{-1}(-1)\right]$
= $\pi \left[\frac{\pi}{2}\right] = \frac{\pi^2}{2}$
$I = \frac{\pi^2}{4}$

29. The given differential equation is,
x dy + y dx = xy dx
$\Rightarrow$ x dy = (x - 1) y dx
$\Rightarrow \frac{1}{y} dy = \left(1 - \frac{1}{x}\right) dx$
$\Rightarrow \int \frac{1}{y} dy = \int \left(1 - \frac{1}{x}\right) dx$
$\Rightarrow$ log |y| = x - log |x| + C
$\Rightarrow$ log |y| - log |x| = x + C
$\Rightarrow$ log |xy| = x + C
$\Rightarrow$ |xyl = e^{x+C} ...(i)
It is given that y(1) = 1 i.e. y = 1 when x = l .Putting x = l and y = 1 in (i), we get
1 = $e^{1+C} \Rightarrow e^0 = e^{1+C} \Rightarrow$ C = -1
Putting C = -1 in (i), we get
$\therefore$ |xy| = e^{x-1}
$\Rightarrow$ xy = $\pm e^{x-1}$
$\Rightarrow y = \pm \frac{1}{x} e^{x-1}$

QuestIQ ACADEMY THINK BEYOND LIMITS. AI POWERED LEARNING CONCEPT MASTERY PERFORMANCE FOCUSED EXCELLENCE DRIVEN Powered by QualCrypt

$\Rightarrow y = \frac{1}{x}e^{x-1}$ or, $y = -\frac{1}{x}e^{x-1}$

But, $y = -\frac{1}{x}e^{x-1}$ is not satisfied by y(1) = 1. Also, $y = \frac{1}{x}e^{x-1}$ is defined for all $x \neq 0$

Hence, $y = \frac{1}{x}e^{x-1}, x \in R$ - {0} is the required solution.

OR

The given differential equation is,

$xe^{\frac{y}{x}} - y + x\frac{dy}{dx} = 0$

$\Rightarrow x\frac{dy}{dx} = y - xe^{\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = \left(\frac{y}{x}\right) - e^{\frac{y}{x}}$

$\Rightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$

$\Rightarrow$ the given differential equation is a homogenous equation.

The solution of the given differential equation is:

Put y = vx

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \left(\frac{vx}{x}\right) - e^{\frac{vx}{x}}$

$\Rightarrow x\frac{dv}{dx} = -e^{v}$

$\Rightarrow \frac{dv}{e^{v}} = \frac{-dx}{x}$

Integrating both the sides we get:

$\Rightarrow \int \frac{dv}{e^{v}} = -\int \frac{dx}{x} + c$

$\Rightarrow -e^{-v} = -\ln|x| + c$

Resubstituting the value of y = vx we get

$\Rightarrow -e^{-\left(\frac{y}{x}\right)} = -\ln|x| + c$

Now, y (e) = 0

$\Rightarrow -e^{-(0)} = -\ln|e| + c$

$\Rightarrow c = 0$

$\Rightarrow -e^{-\left(\frac{y}{x}\right)} = -\ln|x|$

$\Rightarrow y = -x \log(\log|x|)$

30. Let P be the point with position vector $\vec{p} = -\hat{\imath} - 5\hat{\jmath} - 10\hat{k}$ and Q be the point of intersection of the given line and the plane.

We have the line equation as

$\vec{r} = (2\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(3\hat{\imath} + 4\hat{\jmath} + 12\hat{k})$

$\therefore \vec{r} = (2 + 3\lambda)\hat{i} + (-1 + 4\lambda)\hat{j} + (2 + 12\lambda)\hat{k}$

Let the position vector of Q be $\vec{q}$. As Q is a point on this line, for some scalar α, we have

$\Rightarrow \vec{q} = (2 + 3\alpha)\hat{i} + (-1 + 4\alpha)\hat{j} + (2 + 12\alpha)\hat{k}$

This point Q also lies on the given plane, which means this point satisfies the plane equation $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

$\Rightarrow [(2 + 3\alpha)\hat{\mathbf{i}} + (-1 + 4\alpha)\hat{\mathbf{j}} + (2 + 12\alpha)\hat{\mathbf{k}}] \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}} + \hat{\mathbf{k}}) = 5$

$\Rightarrow (2 + 3\alpha)(1) + (-1 + 4\alpha)(-1) + (2 + 12\alpha)(1) = 5$

$\Rightarrow 2 + 3\alpha + 1 - 4\alpha + 2 + 12\alpha = 5$

$\Rightarrow 11\alpha + 5 = 5$

$\Rightarrow 11\alpha = 0$

$\therefore \alpha = 0$

We have $\vec{q} = (2 + 3\alpha)\hat{i} + (-1 + 4\alpha)\hat{j} + (2 + 12\alpha)\hat{k}$

$\Rightarrow \vec{q} = [2 + 3(0)]\hat{i} + [-1 + 4(0)]\hat{j} + [2 + 12(0)]\hat{k}$

$\therefore \vec{q} = 2\hat{i} - \hat{j} + 2\hat{k}$

Using the distance formula, we have

$PQ = \sqrt{(2 - (-1))^2 + ((-1) - (-5))^2 + (2 - (-10))^2}$

$\Rightarrow PQ = \sqrt{3^2 + 4^2 + 12^2}$

$\Rightarrow PQ = \sqrt{9 + 16 + 144}$

$\therefore PQ = \sqrt{169} = 13$

Thus, the required distance is 13 units.

OR

Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\vec{d} \perp \vec{a}, \vec{d} \cdot \vec{a} = 0 \quad \Rightarrow (a_1\hat{i} + a_2\hat{j} + a_3\hat{x})(\hat{i} - \hat{j}) = 0$

$\Rightarrow a_1 - a_2 = 0$ ...(i)

$\vec{d} \perp \vec{b}, \vec{a} \cdot \vec{b} = 0 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(3j - \hat{k}) = 0$

$\Rightarrow 3a_2 - a_2 = 0$ ...(ii)

$\vec{d}\vec{c} = 1 \quad \Rightarrow \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)(7\hat{i} - \hat{k}) = 1$

$\Rightarrow 7a_1 - a_3 = 1$ ...(iii)

Solving equation (i) and (ii) we get $3a_1 - a_3 = 0$ ...(iv)

Again solving equation (iii) & (iv) we get $a_1 = \frac{1}{4}$

From equation (i), $a_1 - a_2 = 0$ or $a_1 = a_2 = \frac{1}{4}$

From equation (ii), $3a_2 - a_2 = 0 \Rightarrow 3 \cdot \frac{1}{4} = a_3 \Rightarrow a_3 = \frac{3}{4}$

Hence, $\vec{d} = \frac{1}{4}\hat{i} + \frac{1}{4}\hat{j} + \frac{3}{4}\hat{k}$

31. Here,it is given that x + 2 y = 8 ...(1)

2x + y = 2 ...(2)

x - y = 1 ...(3)

Line (1) shaded area and origin lie on the same sides of x + 2 y = 8 Corresponding inequations is x + 2y $\leq$ 8 ,Now we have also

Line (2) shaded area and origin lie on the opposite side of 2x + y = 2

The corresponding inequation is 2x + y $\geq$ 2

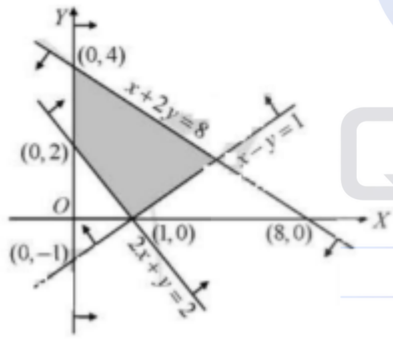
Line (3) shaded area and origin lie on the same side of x - y =1

Corresponding inequation is x - y < 1

shaded area on right side of y - axis.

Corresponding inequation is y > 0

Therefore, the linear constraints are $x \geq 0, y \geq 0, 2x + y \geq 2, x - y \leq 1$ and $x + 2y \leq 8$



Section D

32. The given equations are :

$y^2 = 16ax$ ...(1)

y = 4mx ....(2)

Equation (1) represent a parabola having centre at the origin and vertex along positive x−axis.

Equation (2) represents a straight line passing through the origin and making an angle of 45 with x−axis.

POINTS OF INTERSECTION :

Put y = 4mx in (1), we get

$16m^2x^2 - 16ax = 0$

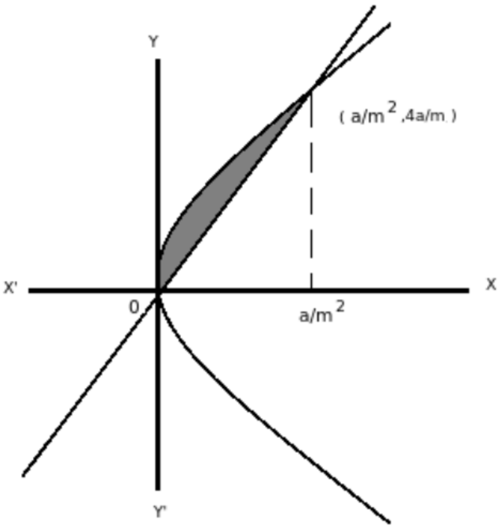
$\Rightarrow 16x\,[m^2x - a] = 0$

$\Rightarrow x = 0; x = \frac{a}{m^2}$

When x = 0; y = 0

When x = $\frac{a}{m^2}$, then y = $\frac{4a}{m}$

QuestIQ ACADEMY
THINK BEYOND LIMITS.
AI POWERED LEARNING
CONCEPT MASTERY
PERFORMANCE FOCUSED
EXCELLENCE DRIVEN
Powered by QualCrypt



Required area =Area under parabola - Area under line

$= 4\sqrt{a}\int_0^{a/m^2}\sqrt{x}dx - 4m\int_0^{a/m^2}xdx$

$= 4\sqrt{\mathrm{a}} \times \frac{2}{3}\left[\mathrm{x}^{\frac{3}{2}}\right]_0^{\frac{\mathrm{a}}{\mathrm{m}^2}} - \frac{4\mathrm{m}}{2}\left[\mathrm{x}^2\right]_0^{\frac{\mathrm{a}}{\mathrm{m}^2}}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3}$

$= \frac{8}{3}\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\frac{a^2}{m^3}$

Now, area = $\frac{a^2}{12}$

So, $\frac{2}{3}\frac{a^2}{m^3} = \frac{a^2}{12}$

$\Rightarrow \mathrm{m}^3 = 8$

$\Rightarrow$ m = 2

33. According to the question, $\vec{a}\times\vec{b} = \vec{c}\times\vec{d}$ ...(i)

and $\vec{a}\times\vec{c} = \vec{b}\times\vec{d}$ ...(ii)

Subtracting Eq. (ii) from Eq. (i),

$(\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) = (\vec{c}\times\vec{d}) - (\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}\times\vec{b}) - (\vec{a}\times\vec{c}) + (\vec{b}\times\vec{d}) - (\vec{c}\times\vec{d}) = \vec{0}$

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c}) + (\vec{b}-\vec{c})\times\vec{d} = \vec{0}$

$\Rightarrow \quad \vec{a}\times(\vec{b}-\vec{c}) - \vec{d}\times(\vec{b}-\vec{c}) = \vec{0}$

$[\because \vec{a}\times\vec{b} = -\vec{b}\times\vec{a}]$

$\therefore \quad (\vec{a}-\vec{d})\times(\vec{b}-\vec{c}) = \vec{0}$

$[\because \vec{a}\neq\vec{d}$ and $\vec{b}\neq\vec{c}$, given]

The cross-product of vectors $\vec{a}-\vec{d}$ and $\vec{b}-\vec{c}$ is a zero vector, so $\vec{a}-\vec{d}$ is parallel $\vec{b}-\vec{c}$.

OR

Let $\vec{\beta} = \lambda\vec{\alpha}$ $\left[\because \vec{\beta}_1 \parallel to\, \vec{\alpha}\right]$

$\vec{\beta}_1 = \lambda(3\hat{i} - \hat{j})$

$= 3\lambda\hat{i} - \lambda\hat{j}$

$\vec{\beta}_2 = \vec{\beta} - \vec{\beta}_1$

$= (2\hat{i} + \hat{j} - 3\hat{k}) - (3\lambda\hat{i} - \lambda\hat{j})$

$= (2-3\lambda)i + (1+\lambda)\hat{j} - 3\hat{k}$

$\vec{\alpha}.\vec{\beta}_2 = 0$ $\left[\because \vec{\beta}_2 \perp \vec{\alpha}\right]$

$3(2-3\lambda) - (1+\lambda) = 0$

$\lambda = \frac{1}{2}$

$\vec{\beta}_1 = \frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}$

$\vec{\beta}_2 = \frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} - 3\hat{k}$

QuestIQ ACADEMY THINK BEYOND LIMITS. AI POWERED LEARNING CONCEPT MASTERY PERFORMANCE FOCUSED EXCELLENCE DRIVEN Powered by QualCrypt

34. L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} - 6\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} + 7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} + 2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$= \begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix} - 6\begin{bmatrix}1+0+4&0+0+0&2+0+6\\0+0+2&0+4+0&0+2+3\\2+0+6&0+0+0&4+0+9\end{bmatrix}$$

$$+ \begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix} + \begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$= \begin{bmatrix}5+0+16&0+0+0&10+0+24\\2+0+10&0+8+0&4+4+15\\8+0+26&0+0+0&16+0+39\end{bmatrix} - \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix} + \begin{bmatrix}7+2&0+0&14+0\\0+0&14+2&7+0\\14+0&0+0&21+2\end{bmatrix}$$

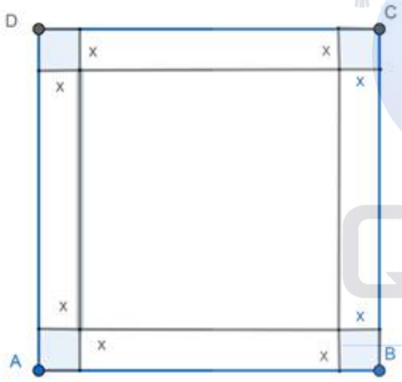
$$= \begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix} - \begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix} = \begin{bmatrix}21-30&0-0&34-48\\12-12&8-24&23-30\\34-48&0-0&55-78\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix}$$

$$= \begin{bmatrix}-9&0&-14\\0&-16&-7\\-14&0&-23\end{bmatrix} + \begin{bmatrix}9&0&14\\0&16&7\\14&0&23\end{bmatrix} = \begin{bmatrix}-9+9&0+0&-14+14\\0+0&-16+16&-7+7\\-14+14&0+0&-23+23\end{bmatrix}$$

$$= \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix} = 0 \text{ (Zero matrix)}$$

= R.H.S. Proved.

35. Given, the Side of the square piece is 18 cms.



Let us consider,

'x' be the length and breadth of the piece cut from each vertex of the piece.

Side of the box now will be (18 - 2x)

The height of the newly formed box will also be 'x'.

Let the volume of the newly formed box is :

$V = (18 - 2x)^2 \times (x)$

$V = 4x^3 - 72x^2 + 324x$ ...(i)

For finding the maximum/ minimum of a given function, we can find it by differentiating it with x and then equating it to zero.

This is because if the function V(x) has a maximum/minimum at a point c then V'(c) = 0.

Differentiating the equation (i) with respect to x:

$\frac{dV}{dx} = \frac{d}{dx}\left[4x^3 - 72x^2 + 324x\right]$

$\frac{dV}{dx} = 12x^2 - 144x + 324$ ...(ii)

[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]

To find the critical point, we need to equate equation (ii) to zero.

$\frac{dV}{dx} = 12x^2 - 144x + 324 = 0$

$x^2 - 12x + 27 = 0$

$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(27)}}{2(1)}$

$= \frac{12 \pm \sqrt{144-108}}{2} = \frac{12 \pm \sqrt{36}}{2}$

QuestIQ ACADEMY THINK BEYOND LIMITS.

$x = \frac{12 \pm 6}{2}$
x = 9 or x = 3
x = 2
[as x = 9 is not a possibility, because 18 - 2x = 18 - 18 = 0]
Now to check if this critical point will determine the maximum area of the box, we need to check with second differential which needs to be negative.
Consider differentiating the equation (ii) with x:
$\frac{d^2V}{dx^2} = \frac{d}{dx}\left[12x^2 - 144x + 324\right]$
$\frac{d^2v}{dx^2} = 24x - 144$ ....(iv)
[Since $\frac{d}{dx}(x^n) = nx^{n-1}$]
Now let us find the value of
$\left(\frac{d^2V}{dx^2}\right)_{x=3}$ = 24(3) - 144 = 72 - 144
= -72
As $\left(\frac{d^2V}{dx^2}\right)_{x=3} = -72 < 0$, so the function V is maximum at x = 3cm
Now substituting x = 3 in 18 - 2x, the side of the considered box:
Side = 18 - 2x = 18 - 2(3) = 18 - 6 = 12cm
Therefore side of wanted box is 12cms and height of the box is 3cms.
Now, the volume of the box is
V = $(12)^2 \times 3 = 144 \times 3 = 432\text{cm}^3$
Hence maximum volume of the box formed by cutting the given 18cms sheet is 432cm^3 with 12cms side and 3cms height.

OR

Let r be the radius, h be the height, V be the volume and S be the total surface area of a right circular cylinder which is open at the top .
Now, given that $V = \pi r^2 h$
$\Rightarrow \quad h = \frac{V}{\pi r^2}$
We know that, total surface area S is given by
$S = 2\pi rh + \pi r^2$
[$\because$Cylinder is open at the top, therefore S= curved surface area of cylinder+area of base]
$\Rightarrow \quad S = 2\pi r\left(\frac{V}{\pi r^2}\right) + \pi r^2$
$\left[\text{put } h = \frac{V}{\pi r^2}, \text{ from Eq. (i)}\right]$
$\Rightarrow \quad S = \frac{2V}{r} + \pi r^2$
On differentiating both sides w.r.t.r, we get
$\frac{dS}{dr} = -\frac{\mathbf{2V}}{r^2} + 2\pi r$
For maxima or minima, put $\frac{dS}{dr} = 0$
$\Rightarrow \quad -\frac{2V}{r^2} + 2\pi r = 0 \Rightarrow V = \pi r^3$
$\Rightarrow \quad \pi r^2 h = \pi r^3 \quad [\because V = \pi r^2 h]$
$\Rightarrow \quad h = r$
Also, $\frac{d^2S}{dr^2} = \frac{d}{dr}\left(\frac{dS}{dr}\right) = \frac{d}{dr}\left(\frac{-\mathbf{2V}}{r^2} + 2\pi r\right)$
$\Rightarrow \quad \frac{d^2S}{dr^2} = \frac{4V}{r^3} + 2\pi$
On putting r=h, we get
$\left[\frac{d^2S}{dr^2}\right]_{r=h} = \frac{4V}{h^3} + 2\pi > 0$ as $h > 0$
Then, $\frac{d^2S}{dr^2} > 0$
Thus,S is minimum.
Hence, S is minimum, when h = r, i.e. when height of cylinder is equal to radius of the base.

Section E

36. i. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.
$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Economics if it is known that he has failed in Mathematics.

Required probability = $P(\frac{E}{M})$

$= \frac{P(E \cap M)}{P(M)} = \frac{\frac{1}{4}}{\frac{7}{20}} = \frac{1}{4} \times \frac{20}{7} = \frac{5}{7}$

ii. Let E denote the event that student has failed in Economics and M denote the event that student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has failed in Mathematics if it is known that he has failed in Economics.

Required probability = P(M/E)

$= \frac{P(M \cap E)}{P(E)} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

iii. Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Mathematics if it is known that he has failed in Economics

Required probability = P(M'/E)

$\Rightarrow P\left(M'/E\right) = \frac{P(M' \cap E)}{P(E)}$

$= \frac{P(E) - P(E \cap M)}{P(E)}$

$= \frac{\frac{1}{2} - \frac{1}{4}}{\frac{1}{2}}$

$\Rightarrow P\left(M'/E\right) = \frac{1}{2}$

OR

Let E denote the event that the student has failed in Economics and M denote the event that the student has failed in Mathematics.

$\therefore P(E) = \frac{50}{100} = \frac{1}{2}$, P(M) = $\frac{35}{100} = \frac{7}{20}$ and P(E ∩ M) = $\frac{25}{100} = \frac{1}{4}$

The probability that the selected student has passed in Economics if it is known that he has failed in Mathematics

Required probability = P(E'/M)

$\Rightarrow P\left(E'/M\right) = \frac{P(E' \cap M)}{P(M)}$

$= \frac{P(M) - P(E \cap M)}{P(M)}$

$= \frac{\frac{7}{20} - \frac{1}{4}}{\frac{7}{20}} \Rightarrow P\left(E'/M\right) = \frac{2}{7}$

37. i. B = {1, 2, 3, 4, 5, 6)

R = {(x, y) : y is divisible by x}

Now, since x is divisible by x

⇒ (x, x) ∈ R

So R is reflexive

Also, here (2, 4) ∈ R, as 4 is divisible by 2

But (4, 2) ∉ R as 2 is not divisible by 4

So R is not symmetric

Now, if (x, y) ∈ R & (y, z) ∈ R

Then (x,z) ∈ R

So R is transitive

Hence R is reflexive and transitive

ii. As A has 2 elements and B has 6 elements

So, number of functions from A to B = 6^2.

iii. ∵ (1, 1) ∉ R

So R is not reflexive

Now, here (1, 2) ∈ but (2, 1) ∉ R

So R is not symmetric

Also (1, 3) ∈ R and (3, 4) ∈ R

But (1, 4) ∉ R

QuestIQ ACADEMY THINK BEYOND LIMITS. AI POWERED LEARNING CONCEPT MASTERY PERFORMANCE FOCUSED EXCELLENCE DRIVEN Powered by QualCrypt

So R is not transitive.

OR

Number of relations

$= 2^{\text{number of element in A} \times \text{ number of element in B}}$

$= 2^{2 \times 6}$

$= 2^{12}$

38. i. R.H.D. of f(x) at x = 1 = $\lim\limits_{h \to 0} \frac{f(1+h)-f(1)}{h}$

$= \lim\limits_{h \to 0} \frac{|1+h-3|-|-2|}{h} = \lim\limits_{h \to 0} \frac{2-h-2}{h}$ = -1

ii. L.H.D. of f(x) at x = 1 = $\lim\limits_{h \to 0} \frac{f(1-h)-f(1)}{-h}$

$= \lim\limits_{h \to 0} \frac{\left[\frac{(1-h)^2}{4} - \frac{3(1-h)}{2} + \frac{13}{4} - 2\right]}{-h}$

$= \lim\limits_{h \to 0} \left[\frac{h^2-2h+1-6+6h+13-8}{-4h}\right]$

$= \lim\limits_{h \to 0} \left[\frac{h^2+4h}{-4h}\right]$ = -1.

iii. Since L.H.D. of f(x) at x = 1

is same as R.H.D. of f(x) at x = 1,

f(x) is differentiable at x = 1.

OR

$$f(x) = \begin{cases} x-3, x \geq 3 \\ 3-x, 1 \leq x < 3 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, x < 1 \end{cases}$$

$[f'(x)]_{x=2} = 0 - 1 = -1$

$[f'(x)]_{x=-1} = \frac{2(-1)}{4} - \frac{3}{2} = -2$

QuestIQ ACADEMY
THINK BEYOND LIMITS.
AI POWERED LEARNING | CONCEPT MASTERY | PERFORMANCE FOCUSED | EXCELLENCE DRIVEN
Powered by QualCrypt