

Section A

1.

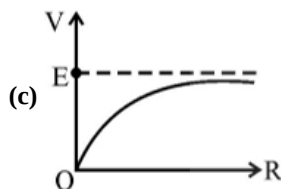
(d) Sample X is undoped while samples Y and Z have been doped with a fifth group and a third group impurity respectively

Explanation:

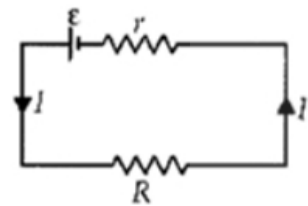
The solution is correct because it accurately identifies the doping characteristics of the semiconductor samples based on their energy band diagrams. In semiconductor physics, undoped silicon has a specific energy band structure, while doping with third group (p-type) or fifth group (n-type) impurities alters this structure. Sample X , being undoped, shows a typical band gap without any significant shifts in the energy levels. In contrast, samples Y and Z exhibit shifts in their energy levels indicative of doping: sample Y shows characteristics of n-type doping (fifth group), while sample Z shows characteristics of p-type doping (third group).

Thus, the assertion in correct option aligns with the expected behavior of the energy band diagrams for the respective doping types.

2.



Explanation:



Current in the circuit, $I = \frac{\varepsilon}{R+r}$

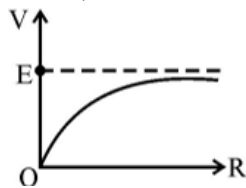
Potential difference across R ,

$$V = IR = \left(\frac{\varepsilon}{R+r} \right) R$$

$$V = \frac{\varepsilon}{1 + \frac{r}{R}}$$

When $R = 0$, $V = 0$

$R = \infty$, $V = \varepsilon$



3. (a) $\frac{f}{x}$

Explanation:

$$u = f + x$$

Using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\text{Or, } \frac{1}{v} - \frac{1}{(f+x)} = -\frac{1}{f}$$

$$\therefore v = -\frac{f(f+x)}{x}$$

So, the magnification $= |m| = \frac{v}{u} = \frac{f}{x}$



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4. (a) Magnetic dipole

Explanation:

Magnetic dipole

5.

- (c) A potential difference appears between the two cylinders when inner cylinder is charged.

Explanation:

When the charge is given to inner cylinder, then an electric field is produced between cylinders which is given by $E = \frac{\lambda}{2\pi\epsilon_0 r}$ and due to this a potential difference is developed between two cylinders.

6. (a) the magnetic field on the tape

Explanation:

A tape is coated with tiny magnet particles. These particles get magnetized when the electric signal passes through them. Thus, a tape recorder records sound in the form of a magnetic field on the tape.

7. (a) Electric motor

Explanation:

Electric motor

8.

- (c) $\chi \propto T^{-1}$

Explanation:

$\chi \propto T^{-1}$

9.

- (b) both light and sound waves

Explanation:

Interference is possible both in light and sound waves.

10. (a) They form closed loops

Explanation:

Electric field lines may not always form closed loops.

11. (a)



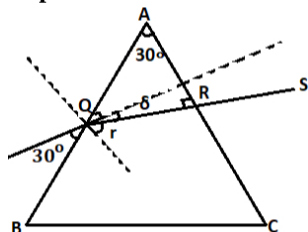
Explanation:

When an alternating voltage is applied across a half wave rectifier, a pulsating voltage appears across the load only during the half cycles of the ac input i.e., only when the diode is forward biased.

12.

- (d) 30°

Explanation:



In $\triangle AQR$

$$\angle A + \angle Q + \angle R = 180^\circ$$

$$30^\circ + 90^\circ - r + 90^\circ = 180^\circ$$

$$r = 30^\circ$$

$$\delta = 90^\circ - 30^\circ - r$$

$$\delta = 90^\circ - 30^\circ - 30^\circ$$

$$\delta = 30^\circ$$

so angle of deviation is 30 degree.

13.

(d) A is false but R is true.

Explanation:

A is false but R is true.

14.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation:

Both A and R are true but R is not the correct explanation of A.

15.

(d) A is false and R is also false

Explanation:

A is false and R is also false

16.

(c) A is true but R is false.

Explanation:

$$\cos \phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

When ω is doubled, power factor ($\cos \phi$) decreases.

So, A is true but R is false.

Section B

17. Here, $v = 6 \times 10^{14}$ Hz, $E_0 = 2 \text{ Vm}^{-1}$

i. Average energy density of the electric field

$$u_E = \frac{1}{4} \epsilon_0 E_0^2 = \frac{1}{4} \times (8.85 \times 10^{-12}) \times 2^2$$

$$= 8.85 \times 10^{-12} \text{ Jm}^{-3}$$

ii. Average energy density of magnetic field

$$u_B = \frac{B_0^2}{4\mu_0} = \frac{1}{4} \times \frac{(E_0/c)^2}{\mu_0} = \frac{1}{4} \frac{E_0^2}{\mu_0 c^2}$$

$$= \frac{1}{4} \times \frac{2^2}{(4\pi \times 10^{-7}) \times (3 \times 10^8)^2}$$

$$= 8.85 \times 10^{-12} \text{ Jm}^{-3}$$

18. i. In stable equilibrium,

$$U = -mB \cos 0^\circ = -mB$$

ii. When the dipole is rotated through 180° , the work done is

$$W = -mB (\cos 180^\circ - \cos 0^\circ)$$

$$= -mB (-1 - 1)$$

$$= +2mB$$

OR

Here $l = 30 \text{ cm} = 0.30 \text{ m}$, $A = 1 \text{ cm}^2 = 10^{-4} \text{ m}^2$, $N = 300$, $I = 0.032 \text{ A}$, $\phi = 2 \times 10^{-6} \text{ Wb}$,

$$n = \frac{N}{l} = \frac{300}{0.30} = 1000 \text{ m}^{-1}$$

Magnetic flux density,

$$B = \frac{\phi}{A} = \frac{2 \times 10^{-6}}{10^{-4}} = 2 \times 10^{-2} \text{ Wb m}^{-2}$$

Magnetising field intensity,

$$H = nI = 1000 \times 0.032 = 32 \text{ A turns m}^{-1}$$

Permeability,

$$\mu = \frac{B}{H} = \frac{2 \times 10^{-2}}{32} = 6.25 \times 10^{-4} \text{ T mA}^{-1}$$

$$\text{Relative permeability, } \mu_r = \frac{\mu}{\mu_0} = \frac{6.25 \times 10^{-4}}{4\pi \times 10^{-7}} = 500$$

$$19. \text{ Here } n_e = 8 \times 10^{13} \text{ cm}^{-3}, n_h = 5 \times 10^{12} \text{ cm}^{-3}, \mu_e = 23,000 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}, \mu_h = 100 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$$

i. As $n_e > n_h$, the semiconductor must be n-type.

ii. Resistivity,

$$\begin{aligned} \rho &= \frac{1}{e(n_e \mu_e + n_h \mu_h)} \\ &= \frac{1}{1.6 \times 10^{-19} (8 \times 10^{13} \times 23 \times 10^3 + 5 \times 10^{12} \times 10^2)} \Omega \text{ cm} \\ &= \frac{1}{1.6 \times 10^{-19} (184 \times 10^{16} + 5 \times 10^{14})} \Omega \text{ cm} \\ &= \frac{1}{1.6 \times 10^{-5} \times 18405} \Omega \text{ cm} = 3.395 \Omega \text{ cm} \end{aligned}$$

20. The symbol b represents impact parameter and θ represents the scattering angle.

i. When $\theta = 0^\circ$ the impact parameter b is large and the α -particle passes almost undeflected.

ii. When $\theta = \pi$ radians, the impact parameter $b = 0$ and the α -particle is reversed back along its original path.

$$21. \oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

There is an inconsistency in the Ampere's Circuital law.

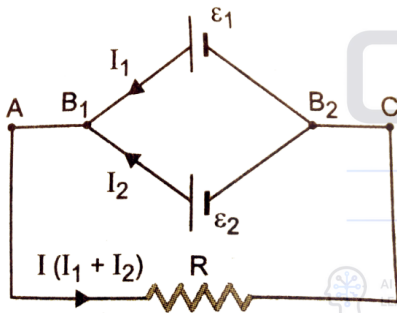
Maxwell modified it as

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + i_d)$$

$$\text{where } i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

Section C

22. As the effective internal resistance of two cells is less than the internal resistance r_1 of one cells so the cell are connected in parallel between the two point R_1 and R_2 . The circuit is the potential difference between point R_1 and R_2 . The potential difference between the terminal of the first cell is



$$V = V_{B_1} - V_{B_2} = \varepsilon_1 - I_1 r_1 \text{ or } I_1 = \frac{\varepsilon_1 - V}{r_1}$$

Potential difference between the terminal of second cell is

$$V = V_{B_1} - V_{B_2} = \varepsilon_2 - I_2 r_2 \text{ or } I_2 = \frac{\varepsilon_2 - V}{r_2}$$

Hence current in external resistance

$$\begin{aligned} I &= I_1 + I_2 = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} \\ &= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) - V \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \\ \text{or } I &= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) - IR \left(\frac{1}{r_1} + \frac{1}{r_2} \right) [\because V = IR] \\ \text{or } I \left[1 + R \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right] &= \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \right) \\ \text{or } \left[\frac{r_1 r_2 + R(r_2 r_1)}{r_1 r_2} \right] I &= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} \\ \text{or } I &= \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{(r_1 \times r_2) + R(r_2 + r_1)} \end{aligned}$$

23. Given that

Initial kinetic energy of photoelectrons is given by $= K_1$

Final kinetic energy of photoelectrons is given by $K_2 = 2K_1$

Wavelength of light changes from λ_1 to λ_2

Let the threshold frequency is ν_0 and work function is ϕ_0

Now, we know that:-

$$\frac{hc}{\lambda} = \phi_0 + KE$$

$$\frac{hc}{\lambda_1} = \phi_0 + K_1 \dots (i)$$

$$\frac{hc}{\lambda_2} = \phi_0 + K_2 \dots (ii)$$

$$K_2 = 2K_1$$

$$\frac{hc}{\lambda_2} = \phi_0 + 2K_1 \dots (iii)$$

$$\frac{2hc}{\lambda_1} = 2\phi_0 + 2K_1 \text{ (eq (i) } \times 2)$$

$$\frac{2hc}{\lambda_1} - \frac{hc}{\lambda_2} = \phi_0$$

$$\Rightarrow \phi_0 = hc \left(\frac{2\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} \right)$$

We know

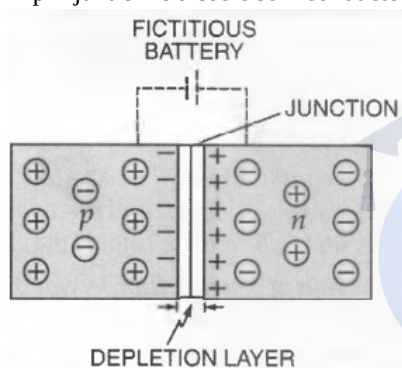
work function is given by $\phi_0 = \frac{hc}{\lambda_0}$

$$\frac{hc}{\lambda_0} = hc \left(\frac{2\lambda_2 - \lambda_1}{\lambda_1 \lambda_2} \right)$$

$$\frac{1}{\lambda_0} = \frac{2\lambda_2 - \lambda_1}{\lambda_1 \lambda_2}$$

$$\lambda_0 = \frac{\lambda_1 \lambda_2}{2\lambda_2 - \lambda_1}$$

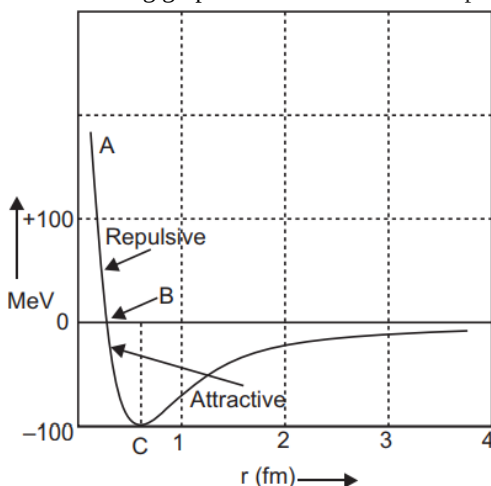
24. A p-n junction is a basic semiconductor device.



When a p-type crystal is placed in contact with n-type crystal so as to form one piece, the assembly so obtained is called p-n junction or junction diode or crystal diode. The surface of contact of p and n-type crystals is called junction. In the p-section, holes are the majority carriers; while in n-section, the majority carriers are electrons. Due to the high concentration of different types of charge carriers in the two sections, holes from p-region diffuse into n-region and electrons from n-region diffuse into p-region. In both cases, when an electron meets a hole, the two cancel the effect of each other and as a result, a thin layer at the junction becomes devoid of charge carriers. This is called the depletion layer as shown in Fig. —

- When a p-n junction is forward biased, the width of the depletion layer decreases. As a result, it offers low resistance during forward bias.
- When a p-n junction is reverse biased, the width of the depletion layer increases. As a result, it offers high resistance during reverse bias.

25. The following graph shows the variation of potential energy with the separation of nucleons



- Part BC of the graph shows the attractive force.
- Part AB of the graph shows the repulsive force.

The characteristic features of the nuclear force are as under:

1. Nuclear forces are attractive and stronger than the electrostatic force.
2. Nuclear forces are charge-independent and short range forces.

26. i. Energy of the ground state ($n = 1$) = – (ionization energy) = –13.6 eV

The wavelength of the incident radiation, $\lambda = 975 \text{ \AA}$

$\therefore$ The energy of the incident photon = hc/λ

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{975 \times 10^{-10} \times 1.6 \times 10^{-19}} = 12.75 \text{ eV}$$

Let electron is excited to n th orbit,

$$\Rightarrow 12.75 = 13.6 \left(\frac{1}{1^2} - \frac{1}{n^2} \right)$$

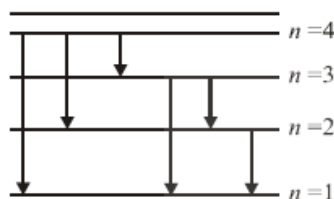
$$\Rightarrow n = 4$$

The quantum transitions to the less excited states gives six possible lines as follows:

$n = 4$: ($4 \rightarrow 3$), ($4 \rightarrow 2$), ($4 \rightarrow 1$)

$n = 3$: ($3 \rightarrow 2$), ($3 \rightarrow 1$)

$n = 2$: ($2 \rightarrow 1$)



ii. The longest wavelength emitted is for the transitions ($4 \rightarrow 3$) where energy difference is minimum.

$$E_{\min} = (E_4 - E_3) = 13.6 \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = 0.661 \text{ eV}$$

$$\begin{aligned} \text{Thus } \lambda_{\max} &= \frac{hc}{E_{\min}} \\ &= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{0.661 \times 1.6 \times 10^{-19}} \text{ m} \\ &\approx 18807 \text{ \AA} \end{aligned}$$

27. Sound waves can propagate only through a medium. Thus, even though both the situations may correspond to the same relative motion (between the source and the observer), they are not identical physically since, the motion of the observer, relative to the medium is different in the two situations. Hence, we cannot expect Doppler formulas for sound to be identical for (a) and (b). For light waves in vacuum, there is clearly nothing to distinguish between two cases given. Here only the relative motion between the source and the observer counts and the relativistic Doppler formula is the same for both cases.

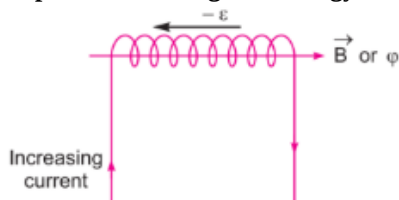
For light propagation in a medium, once again like for sound waves, the two situations are not identical and we should expect the Doppler formulas for this case to be different for the two situations (a) and (b).

28. Using formula, $|\epsilon| = L \frac{dI}{dt}$

If $\frac{dI}{dt} = 1 \text{ A/s}$, then $L = |\epsilon|$

Self inductance of the coil is equal to the magnitude of induced emf produced in the coil itself when the current varies at rate 1 A/s.

Expression for magnetic energy:



When a time varying current flows through the coil, back emf ($-\epsilon$) produces, which opposes the growth of the current flow. It means some work needs to be done against induced emf in establishing a current I . This work done will be stored as magnetic potential energy.

For the current I at any instant, the rate of work done is

$$\frac{dW}{dt} = (-\epsilon)I$$

Only for inductive effect of the coil $|\epsilon| = L \frac{dI}{dt}$

$$\therefore \frac{dW}{dt} = L \left(\frac{dI}{dt} \right) I \Rightarrow dW = LI dI$$

From work-energy theorem,

$$dU = LI dI$$

$$\therefore U = \int_0^I LI dI = \frac{1}{2} LI^2$$

OR

i. In the one revolution, change of area,

$$dA = \pi l^2$$

$\therefore$ Change of magnetic flux in one revolution of the rod,

$$d\phi_B = \vec{B} \cdot d\vec{A} = BdA \cos 0^\circ = B\pi l^2$$

(Given, magnetic field intensity, $\vec{B}$ is parallel to change in area, $d\vec{A}$)

If period of revolution is T,

$$\text{a. Induced emf (e)} = \frac{d\phi}{dt} = \frac{B\pi l^2}{T} = B\pi l^2 \nu \quad \left(\because \nu = \frac{1}{T} \right)$$

b. Induced current in the rod,

$$I = \frac{e}{R} = \frac{\pi \nu B^2 l^2}{R}$$

(Given R = resistance of the rod)

ii. Magnitude of force acting on the rod,

$$|\vec{F}| = |I(\vec{l} \times \vec{B})| = BIl \sin 90^\circ = \frac{\pi \nu B^2 l^3}{R}$$

The external force required to rotate the rod opposes the Lorentz force acting on the rod, i.e external force acts in the direction opposite to the Lorentz force.

iii. Power required to rotate the rod,

$$P = \vec{F} \cdot \vec{v} = Fv \cos 0^\circ = \frac{\pi \nu B^2 l^3 v}{R}$$

Section D

29. Read the text carefully and answer the questions:

Microwave oven: The spectrum of electromagnetic radiation contains a part known as microwaves. These waves have frequency and energy smaller than visible light and wavelength larger than it. What is the principle of a microwave oven and how does it work? Our objective is to cook food or warm it up. All food items such as fruit, vegetables, meat, cereals, etc., contain water as a constituent. Now, what does it mean when we say that a certain object has become warmer? When the temperature of a body rises, the energy of the random motion of atoms and molecules increases and the molecules travel or vibrate or rotate with higher energies. The frequency of rotation of water molecules is about 2.45 gigahertz (GHz). If water receives microwaves of this frequency, its molecules absorb this radiation, which is equivalent to heating up water. These molecules share this energy with neighbouring food molecules, heating up the food. One should use porcelain vessels and non-metal containers in a microwave oven because of the danger of getting a shock from accumulated electric charges. Metals may also melt from heating. The porcelain container remains unaffected and cool, because its large molecules vibrate and rotate with much smaller frequencies, and thus cannot absorb microwaves. Hence, they do not get eaten up. Thus, the basic principle of a microwave oven is to generate microwave radiation of appropriate frequency in the working space of the oven where we keep food. This way energy is not wasted in heating up the vessel. In the conventional heating method, the vessel on the burner gets heated first and then the food inside gets heated because of transfer of energy from the vessel. In the microwave oven, on the other hand, energy is directly delivered to water molecules which is shared by the entire food.

(i) (c) less than visible light

Explanation:

Microwaves have frequency and energy smaller than visible light and wavelength larger than it.

(ii) (d) the energy of the random motion of atoms and molecules increases

Explanation:

When the energy of the random motion of atoms and molecules of a substance increases and the molecules travel or vibrate or rotate with higher energies, the substance becomes hot.

(iii) (b) 2.45 GHz

Explanation:

The frequency of rotation of water molecules is about 2.45 gigahertz.

OR

(d) Energy is directly delivered to water molecules which is shared by the entire food

Explanation:

In the conventional heating method, the vessel on the burner gets heated first and then the food inside gets heated because of transfer of energy from the vessel. In the microwave oven, on the other hand, energy is directly delivered to water molecules which is shared by the entire food.

- (iv) (c) Because of the danger of getting a shock from accumulated electric charges

Explanation:

One should use porcelain vessels and non-metal containers in a microwave oven because of the danger of getting a shock from accumulated electric charges. Metals may also melt from heating. The porcelain container remains unaffected and cool, because its large molecules vibrate and rotate with much smaller frequencies and thus cannot absorb microwaves. Hence, they do not get heated up.

30. i. There are two plates A and B having surface charge densities, $\sigma_A = 17.0 \times 10^{-22} \text{ C/m}^2$ on B, respectively. According to Gauss' theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$E_I = E_A + E_B = \frac{\sigma}{2\epsilon_0} + \left(-\frac{\sigma}{2\epsilon_0}\right) = 0$$

- ii. The electric field in region III is also zero.

$$E_{III} = E_A + E_B = \frac{\sigma}{2\epsilon_0} + \left(-\frac{\sigma}{2\epsilon_0}\right) = 0$$

- iii. In region II or between the plates, the electric field.

$$E_{II} = E_A - E_B = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0}$$

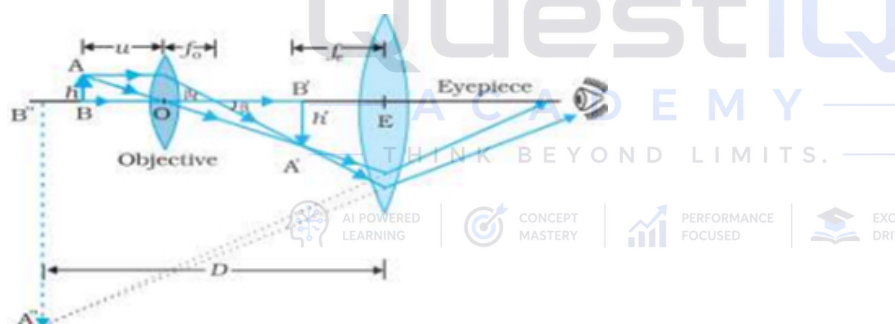
$$= \frac{\sigma(\sigma_A \text{ or } \sigma_B)}{\epsilon_0} = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$

$$E = 1.9 \times 10^{-10} \text{ NC}^{-1}$$

- iv. Since electric field due to an infinite-plane sheet of charge does not depend on the distance of observation point from the plane sheet of charge. So, for the given distances, the ratio of E will be 1 : 1.
- v. In order to estimate the electric field due to a thin finite plane metal plate, we take a cylindrical cross-sectional area A and length 2r as the Gaussian surface.

Section E

31. i.



Linear magnification due to the objective is

$$\tan \beta = \left(\frac{h}{f_o}\right) = \left(\frac{h'}{L}\right)$$

$$m_o = \frac{h'}{h} = \frac{L}{f_o}$$

Here L is the distance between the second focal point of the objective and the first focal point of the eyepiece.

Linear magnification due to eyepiece is

$$m_e = \left(1 + \frac{D}{f_e}\right)$$

Thus, the total magnification is, $m = m_o \times m_e$

$$m = \frac{L}{f_o} \left(1 + \frac{D}{f_e}\right)$$

Hence the expression for the total magnification for the final image to be formed at the near point is $m = \frac{L}{f_o} \left(1 + \frac{D}{f_e}\right)$.

- ii. Given:

$$u_o = -1.5 \text{ cm}$$

$$f_o = 1.25 \text{ cm}$$

$$f_e = 5 \text{ cm}$$

$$D = 25 \text{ cm}$$

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0}$$

$$\frac{1}{v_0} = \frac{1}{1.25} - \frac{1}{1.5} = \frac{2}{15}$$

$$v_0 = \frac{15}{2} \text{ cm}$$

$$|m_0| = \frac{v_0}{u_0} = \frac{15}{2} \times \frac{1}{1.5} = 5$$

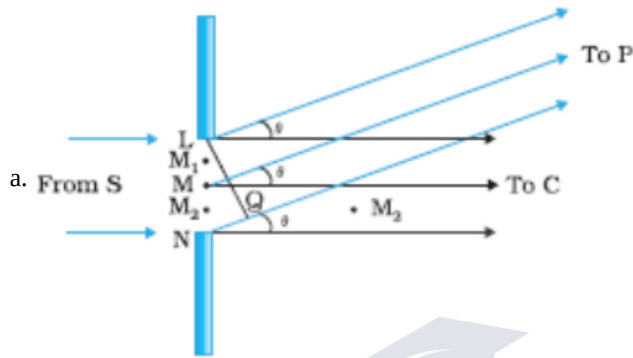
$$|m_e| = \left(1 + \frac{D}{f_e}\right)$$

$$|m_e| = 1 + \frac{25}{5} = 6$$

$$m = m_0 \times m_e = 5 \times 6 = 30$$

hence, the magnifying power of the microscope is 30

OR



From diagram path difference between the waves from L and N = $a \sin \theta$

When first minimum is obtained at P then path difference = λ

[imagine the slit be divided into two halves, for each wavelets from first half of the slit has a corresponding wavelet from second half of the slit differing by a path of $\frac{\lambda}{2}$ and cancel each other]

Condition for first minimum

$$\therefore \lambda = a \sin \theta$$

$$\text{b. } \beta_{\text{cm}} = \frac{2\lambda D}{d}$$

i. As we know that wave length (λ) of red light is more than yellow light.

$$\therefore \lambda_{\text{red}} > \lambda_{\text{yellow}}$$

$$\text{So, } \therefore \beta_{\text{red}} > \beta_{\text{yellow}}$$

Hence, the linear width of the central maximum will increase if monochromatic yellow light is replaced with the red light.

ii. If the distance between the slit and screen (d) is increased then also the linear width of the central maximum will increase.

$$\text{c. } 10 \frac{\lambda}{d} = 2 \frac{\lambda}{a}$$

$$a = \frac{d}{5} = 0.2 \text{ mm}$$



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LEARNING



CONCEPT
MASTERY

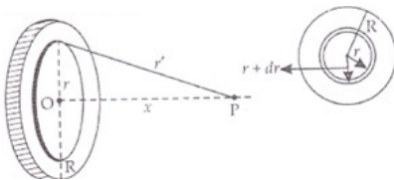


PERFORMANCE
FOCUSED



EXCELLENCE
DRIVEN

32. Consider a point P on the axis perpendicular to the plane of disc and at distance x from the center O of the disc as shown in the figure.



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Now consider a ring of radius r of thickness dr on a disc of radius R, as shown in the figure, Let us consider disc is divided into a large number of rings, Again let the charge on the ring is dq then potential dV due to ring at P, will be

$$dV = \frac{k dq}{r'} \left[\because r' = \sqrt{r^2 + x^2} \right]$$

dq is the charge on the ring = σ area of ring

$$= \sigma \cdot [\pi(r + dr)^2 - \pi r^2]$$

$$dq = \sigma \cdot \pi [r^2 + dr^2 + 2rdr - r^2]$$

Because dr is small, therefore, dr^2 is negligible.

$$\therefore dq = \sigma \pi (2rdr) = 2\pi r \sigma \cdot dr$$

$$\therefore dV = \frac{k \cdot 2\pi r \sigma dr}{\sqrt{(r^2 + x^2)}}$$

So the potential due to charged disc

$$\int_0^V dV = \int_0^R \frac{k \cdot 2\pi\sigma dr}{\sqrt{r^2 + x^2}}$$

$$V = k \cdot 2\pi\sigma \cdot \int_0^R \frac{r dr}{(r^2 + x^2)^{1/2}} = k \cdot \pi\sigma \int_0^R r \cdot (r^2 + x^2)^{-1/2} 2dr = \frac{k\pi\sigma [\sqrt{r^2 + x^2}]_0^R}{1/2}$$

$$= 2\pi k\sigma \left[(R^2 + x^2)^{1/2} - x \right] = \frac{2\pi\sigma}{4\pi\epsilon_0} \left[(R^2 + x^2)^{1/2} - x \right]$$

$$[\because \pi R^2 \sigma = Q(\text{charge on disc}) \sigma = \frac{Q}{\pi R^2}]$$

$$= \frac{\pi 2R^2 \sigma}{4\pi\epsilon_0 R^2} [\sqrt{R^2 + x^2} - x] \text{ thus the potential due to a disc is given by,}$$

$$V = \frac{2Q}{4\pi\epsilon_0 R^2} [\sqrt{R^2 + x^2} - x]$$

OR

- a. Consider a parallel plate capacitor which is connected across a battery. The electrons are transferred from the negative terminal of the battery to the metallic plate connected to the negative terminal and acquires a negative charge. Similarly, the electrons move from the second plate to the positive terminal of the battery and acquire a positive charge. This process continues until the potential difference between the two plates becomes equal to the potential difference between the terminals of the battery. Thus, the charge is developed on the capacitor.

Let 'dW' be the work done by the battery in increasing the charge on the capacitor is given by, having the charge q and potential V is:

$$dW = V dq$$

$$\text{where } V = \frac{q}{C}$$

$$\therefore dW = \frac{q}{C} dq$$

Total work done in charging up the capacitor is given by,

$$W = \int dW = \int_0^Q \frac{q}{C} dq$$

$$\therefore W = \frac{Q^2}{2C}$$

Hence total energy stored in the plates of the capacitor is given by, $W = \frac{Q^2}{2C} = \frac{1}{2} CV^2 = \frac{1}{2} QV$

- b. Charge on the plates of the capacitor is given by $q = CV$

When uncharged capacitor of same capacitance is connected to the charged capacitor, sharing of charges takes place between the two capacitors till both the capacitors acquire same potential $\frac{V}{2}$

Energy stored in the combination of capacitors is given by,

$$U_2 = \frac{1}{2} C \left(\frac{V}{2}\right)^2 + \frac{1}{2} C \left(\frac{V}{2}\right)^2 = \frac{CV^2}{4}$$

Energy stored by a single capacitor before connecting is given by, $U_1 = \frac{1}{2} CV^2$

Ratio of energy stored in the combination to that in the single capacitor is given by,

$$\frac{U_2}{U_1} = \frac{CV^2/4}{CV^2/2} = 1 : 2, \text{ Hence these are required results.}$$

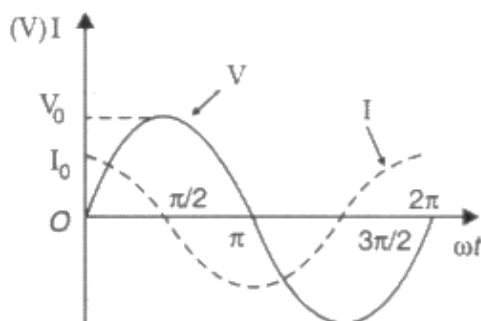
33. i. $V = V_0 \sin \omega t, I = \frac{Q}{C}$

A.C. source containing capacitor: Let a source of alternating e.m.f. $V = V_m \sin \omega t$ be connected to a capacitor of capacitance C only.

$$I = \frac{dQ}{dt}$$

$$I_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)}$$

$$I = I_0 \sin \left(\omega t + \frac{\pi}{2} \right)$$



ii. $X_C = \frac{1}{2\pi fC} = 212.3 \Omega$

$$Z = \sqrt{R^2 + X_C^2} = 291.5 \Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{220}{291.5} = 0.755 \text{ A}$$

$$V_R (\text{rms}) = 151 \text{ V}$$

$$V_C (\text{rms}) = 160.3 \text{ V}$$

Two voltages are out of phase, hence they are added vectorially.

OR

i. Device X is a capacitor.

As the current is leading voltage by $\frac{\pi}{2}$ radians and it happens only then when an ac source is connected with a pure capacitive circuit.

ii. Curve A represents power, Curve B represents voltage and Curve C represents current.

$$\text{As, } V(t) = V_0 \sin \omega t$$

$$\text{Current, } I(t) = I_0 \cos \omega t, \text{ with } I_0 = \frac{V_0}{X_C} \text{ (} X_C \text{ being capacitive reactance)}$$

As, in the case of capacitor,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ [current is leading the voltage]}$$

$$\text{Average power, } P = V(t)I(t) = \frac{V_0 I_0}{2} \cos \phi$$

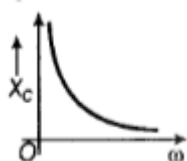
where, ϕ = phase difference

iii. As, X_C = capacitive reactance = $\frac{1}{C\omega}$

where ω is angular frequency and C being capacitance of the capacitor.

So, reactance or impedance decreases with increase in frequency.

Graph of X_C versus ω is shown below:



iv. For a capacitor fed with an AC supply,

$$V = \frac{q}{C} \text{ or } q = CV = CV_0 \sin \omega t$$

$$\therefore I = \frac{dq}{dt} = V_0 \omega C \cos \omega t = \frac{V_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right), \text{ since } \omega C = \frac{1}{X_C}$$



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